

1963

Computer application to cardiovascular disease

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COMPUTER APPLICATION TO CARDIOVASCULAR DISEASE

by

Charles Lee Townsend

A Dissertation Submitted to the
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1963

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I. INTRODUCTION

A. Fundamental Concept of Electrocardiography (1)

Since the latter part of the 19th century it has been known that a measurable amount of electric current is associated with activity of the heart. In 1903, Einthoven's work with a string galvanometer stimulated a sudden increase in both clinical and experimental studies of electrocardiography. With the incorporation of other principles, such as vacuum-tube amplification, this type of galvanometer continues to be most usefully employed to obtain electrocardiographic patterns.

In order to obtain an electrocardiogram, one connection, for example, may be made to the right arm (RA) and another connection made to the left arm (LA) of the subject. Electrodes connected to these parts of the body constitute lead I. Other leads may be recorded by connecting the electrodes to different parts of the body. Current from the heart, conducted through the galvanometer by means of the electrode connections enable one to record the waveform of a cardiac cycle. The human body, by virtue of the chemical nature of its fluids, is essentially a volume conductor. Thus, current generated in any part of the body can reach any other part. By considering the human body as a volume conductor and the electric impulses originating in the heart as a source of potential differences, the magnitude and direction of the potential produced may be measured. The ECG is simply the measurement of the heart's electrical activity.

The typical or normal electrocardiogram of a cardiac cycle is represented diagrammatically in Figure 1. It consists of a series of waves arbitrarily designated by Einthoven as the P wave, the QRS complex, and the

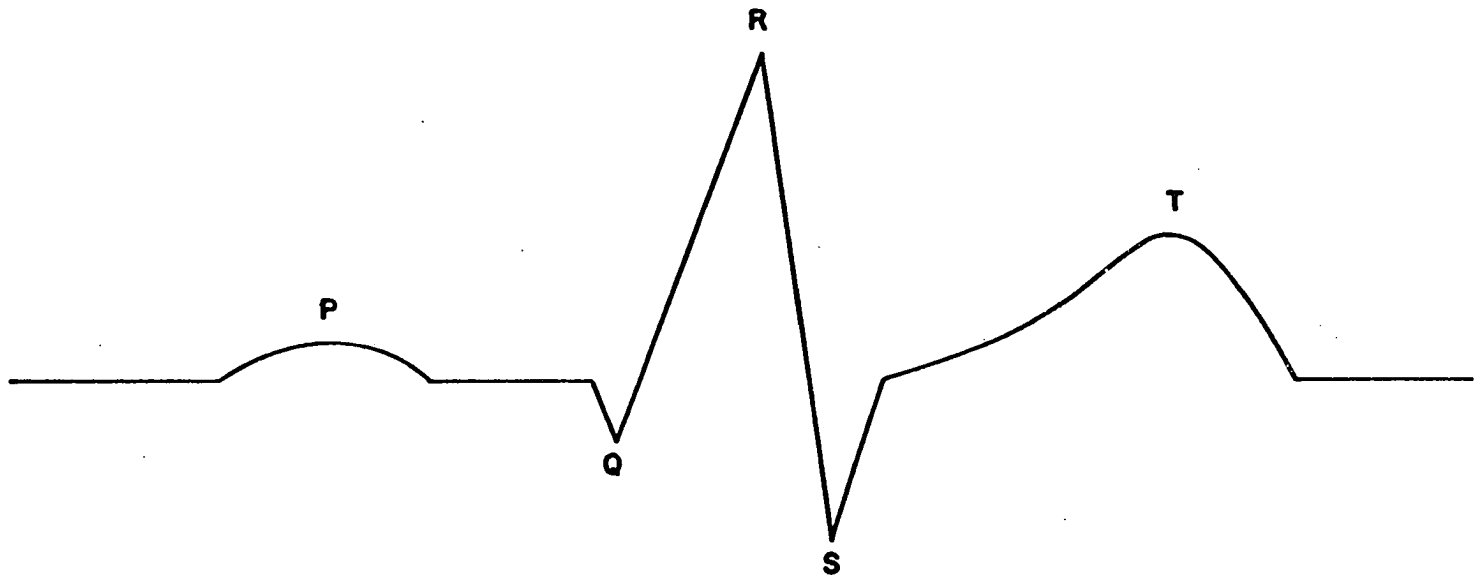


Figure 1. Waves of the electrocardiogram

T wave.

It is known that just before each beat of the heart, the pacemaker, a node of specialized muscle and nerve tissue situated in the wall of the upper right chamber of the heart, discharges a small electric current. This spreads through muscle fibers in the auricles, or upper chambers of the heart. These fibers contract, forcing blood into the ventricles, the lower chambers. The current passes through conducting fibers to spread over thick ventricular walls, and these chambers, too, contract. The right ventricle forces blood to the lungs for oxygen, the left ventricle pushes fresh blood through the rest of the body. With electrodes in place, the heart's minute electric currents are detected and amplified to obtain a complete electrocardiographic waveform. Three major components comprise the resulting curve: the P wave indicates how the auricles are electrically stimulated; the QRS segment indicates how the electrical wave sweeps over the ventricles; the T wave indicates how the ventricles are recharged.

The study of electrocardiographic patterns is basically one of noting amplitude and time intervals for the various segments which comprise the total waveform. Characteristic patterns have been found for a variety of heart diseases. Since the waveform consists essentially of three segments, particular interest is centered around the P wave, the QRS complex, and the T wave. It has become apparent through the years that a diagnosis of cardiac disease is possible by proper interpretation of the electrocardiogram.

An examining cardiologist usually measures numerous parameters associated with the normal electrocardiogram shown in Figure 1. Typical

parameters would likely include the duration of each segment comprising the waveform and the amplitudes of the various waves. Other measurements are made of particular intervals that can be associated with the ECG; the RR interval and the PR interval are just two examples. In order to extract the required information for a particular subject, there are twelve leads normally employed. These measurements are taken from electrodes placed at different points on the body to obtain a total of twelve different records for a complete electrocardiogram.

The present concept of the theory of electrocardiography is based upon data collected from many sources. Some of the main sources of information are:

1. Clinical data collected on patients during life and correlated with information found from autopsy.
2. Physiologic observations on the intact hearts of experimental animals.
3. Study of isolated muscle strips.
4. Studies on the giant axon of the squid, as well as observations made on other nerves by neurophysiologists.
5. Studies on the large one cell plant, such as the *Nitella flexilis*.

From such studies as these listed above it has been possible to begin to classify cardiac diseases from amplitude and time interval observations of the total waveform as well as its individual segments. Consideration of such variables as age, weight, and general physical well being of the patient afford important additional clinical data to be kept in mind.

B. Purpose

The purpose of this dissertation is to report on the results of one investigation being conducted in cooperation with the Heart Station at Iowa Methodist Hospital of Des Moines. This study was conducted with the specific notion of making an automatic diagnosis of heart disease by utilizing a digital computer to "recognize" the electrocardiogram. An investigation of the use of correlation techniques in the diagnosis of the electrocardiogram is the basic objective of this dissertation. It should be emphasized that correlation diagnosis is basically a shape recognition method entailing a normalization procedure in both amplitude and time.

Studies of interest to the engineer which pertain to electrocardiography are now being conducted by various professional-technical teams primarily in the following areas:

1. Automatic recording.
2. Spacial vectorcardiography.
3. Automatic diagnosis by digital computer.
4. Electrical models of the heart.

Iowa Methodist Hospital of Des Moines has pursued during the past two years a research effort aimed at determining the usefulness of digital computers in the practice of medicine. This study has demonstrated the use of computers as an aid in diagnosis in the field of pediatric cardiology.

Recently, an investigation of automatic interpretation of the clinical electrocardiogram was undertaken as a project for the Heart Station of Iowa Methodist Hospital. For purposes of research by the Heart Station, it is highly desirable to acquire an electrocardiogram on each patient, for

purposes of classification, diagnosis, and statistical studies. This study is a part of a continuing long range research effort being conducted at this facility in an attempt to provide the physician with an invaluable assistant through the incorporation of digital computer techniques as an aid in medical diagnosis.

A number of authors have discussed various possibilities, techniques, and methods for obtaining a machine diagnosis from the electrocardiographic waveform. Some of these authors have considered the correlation technique a valid method of extracting information from the electrocardiogram. It has been pointed out that a correlation technique will permit a greater realization of the information available than that normally contained in the conventional clinical parameters. Through the use of correlation methods, it has been suggested that a reduction in the number of leads required, per patient, should be possible, since a correlation technique yields the same information on the exact nature of the wave shape, as well as the clinical parameters that are usually recorded. No comprehensive study has been reported in which the correlation techniques have been utilized to make a machine diagnosis from the entire waveform of the electrocardiogram. Considerable effort has been expended in attempting to realize a diagnosis of the electrocardiogram with a digital computer by suggesting that the waveform could be separated into three principle segments and applying the correlation methods available to each of these three separated waves.

This dissertation considers the distinct possibility of obtaining a machine diagnosis by utilization of the entire waveform of the electrocardiogram. The complete waveform of only one lead, namely, V_6 , is to be

investigated thoroughly by incorporation of correlation techniques in an attempt to perform a diagnosis with a digital computer. The precordial chest lead, V_6 , is placed on the side of the chest cavity under the left arm. An electrical heart activity is then measured between this electrode and a reference point placed at the neutral of the wye formed by three resistors placed between the neutral point and the three electrodes placed on the left wrist, right wrist, and left leg. Through the use of one lead, in contrast to the total of twelve leads now required for a complete clinical electrocardiogram, it is anticipated that a diagnosis of heart disease can be accomplished with a machine by a pattern recognition process. Hopefully, it is felt that the technique to be demonstrated will indicate considerable promise for simplification of the automatic recording methods presently proposed.

The results of this investigation will answer the following questions:

1. Is it possible to make a diagnosis of heart disease through a pattern recognition scheme incorporating the use of a digital computer?
2. Is the method demonstrated one which lends itself to practical application?
3. How does this method compare with other techniques?

II. REVIEW OF LITERATURE

Within the last three or four years, a considerable effort has been made to utilize programmed digital computers in the interdisciplinary field of Medical Electronics. The use of electronic computers in medical data processing as an aid in diagnosis, for current information retrieval, and in medical record keeping was discussed by Ledley and Lusted (2) in January of 1960. In October 1960 Ledley (3) discussed the specific use of electronic computers in making a medical diagnosis. In both of these papers, the notion of making a medical diagnosis on the basis of a disease-symptom complex requiring a calculation for a conditional probability was presented. Bayes' (4) theorem offered important information concerning the composition of the conditional probability in the disease-symptom complex and was the recommended approach in making a diagnosis by machine.

In July 1961, Warner, Toronto, Veasey, and Stephenson (5) presented a paper giving a mathematical approach to medical diagnosis of congenital heart disease in which a programmed digital computer was used to aid in the lengthy calculations required. This work was direct application of the use of Bayes' theorem for calculating the conditional probability of having a specific cardiac disease, given a symptom complex. The diagnostic results obtained by this technique compared favorably with those made by practicing physicians. Brodman (6) has reported on work done with a data-processing machine programmed to simulate what is postulated to be the operation of a physician's mind when he makes a diagnostic decision. This system derived data from which the diagnostic significance of complaints, which denotes a measure of the probability that a patient making a complaint

has a particular disease, could be ascertained. The programmed computer interpreted patient's medical histories with such discrimination that it identified the patient's disease as often as did a physician interpreting the same data. Balm (7) has published, as a thesis, the results of a study being conducted through Iowa Methodist Hospital of Des Moines. This thesis, entitled "Medical Diagnosis On A Digital Computer Using Probability Techniques", utilized the same method as Warner and his associates, introducing, however, various modifications of the disease-symptom matrix. The results proved again the value of computer techniques in obtaining a valid diagnosis of heart disease. Gustafson (8,9,10) has reported in a number of papers about the effort being conducted at Iowa Methodist Hospital in Des Moines. A program, currently in operation at Iowa Methodist has been described, which duplicates the techniques and diagnostic logic currently employed by electrocardiologists. This program uses a digital computer for implementation of this logic, but uses input data obtained by manual measurements. This effort has been directed at obtaining an automatic interpretation of the electrocardiogram in pediatric patients. In July 1963, at a conference on data processing in biology and medicine at the University of Rochester, Gustafson told the conference that it is essential that today's practicing physician understand what a digital computer can do, and even more important, what it can not do. He went on to indicate that the computer will not replace the physician, but pointed out that it can provide him with an invaluable assistant.

One phase of the current lively interest in artificial intelligence is that of the use of programmed digital computers as general pattern classification and recognition devices. Steinberg, Abraham, and Caceres (11)

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One phase of the current lively interest in artificial intelligence is that of the use of programmed digital computers as general pattern classification and recognition devices. Steinberg, Abraham, and Caceres (11)

have developed a computer program for automatically recognizing and measuring the clinically useful parameters from an electrocardiogram waveform. The program permits determination of amplitude of P, Q, R, S and T waves, ST and PQ segments, and QT and RR intervals. This paper deals with one phase of a project directed at the development of an automated system to aid in the diagnosis of heart disease.

A computer pattern recognition system is proposed by Stark, Okajima and Whipple (12) using programmed digital computers to classify and recognize the electrocardiographic waveform. This proposed experimental data processing system centers upon the use of multiple adaptive matched filters that classify normalized signals. A technique of spatial vectorcardiography is outlined in this proposal to obtain three relatively orthogonalized components. The magnitude of the spatial vector is obtained and then is normalized and split into three parts. The three time and amplitude normalized sectors are handled separately in the suggested adaptive pattern classification process. This paper illustrates the background of the application of computer pattern recognition techniques to classification and automatic diagnosis of clinical electrocardiograms. Mericle (13) has conducted a careful study investigating the possibilities for diagnosis through application of correlation methods by an automatic computer system. The characterization of the electrocardiogram waveform as a series of single pulses, resulting from the various actions of the heart is postulated and the finite correlation functions are computed for these individual pulses. This effort was conducted in conjunction with Iowa Methodist Hospital in order to determine the feasibility of using a pattern recognition scheme as an economical method on a routine basis. It was pointed out

that the techniques outlined constituted a good framework from which to build up the studies of the clinical electrocardiogram.

In the completely automated computer pattern recognition system it is mandatory to know the spectral content of the electrocardiographic waveform in order to determine an acceptable sampling rate of the input data. Thompson (14) has described a method of analysis applied to the electrocardiogram primarily to determine the bandwidth requirements for electrocardiographic amplifiers. The records were also studied with the hope that something of clinical significance might be found. It was pointed out, however, that one must go above fifty-one cycles per second as an upper limit in order to pass a complete electrocardiogram. The lower limit is not determined in this study.

In order to classify as well as to recognize a physical waveform pattern, the interpretation and application of statistical analysis for random physical phenomena is necessary. A number of authors have discussed mathematical and statistical concepts deemed important for application to many physical problems. Particular details on interpreting and applying probability density functions, correlation functions, and power spectral density functions to many problems, including biomedical research, have been discussed by Bendat (15). Brown and Nilsson (16) as well as Truxal (17) include in their books a discussion of the mathematical and statistical concepts one should consider for a comprehensive study of the random physical problem. An acceptable sampling rate for the electrocardiographic waveform can be obtained from a study of Goldman (18) with the utilization of the sampling theorem in the time domain.

No matter what formalism is used to view a given communication or

detection situation, the statistical considerations involved lead usually to some form of correlation or matched filtering as a part of the set of operations that will perform the desired function most efficiently. In a tutorial survey, Turin (19), has attempted to unify the various notions of the separate aspects of the field of matched filters. In this introductory treatment, an attempt has been made to provide engineering insight into the properties of matched filters and some possible forms of matched filters. Also included in this treatment is a discussion of where matched filters arise and matched filter synthesis and signal specification.

III. INVESTIGATION

A. Theoretical Considerations

A considerable amount of effort has been expended by the professional-technical team associated with Iowa Methodist Hospital in the field of machine medical diagnosis. This work has been conducted primarily in the field of congenital heart disease although some basic work has been done with thyroid disease and abdominal pain. This effort to date has included a differential diagnosis technique conducted in conjunction with the general machine diagnostic problem. Logically, the general machine diagnostic study should include an examination of the pattern recognition problem associated with the clinical electrocardiogram.

An immediate objective of the electrocardiographic studies is the processing of a large enough amount of data to develop standards for use in future data processing techniques. Presently, a manual technique is in use, which utilizes the measurements of a technician to operate on the normal clinical parameters and furnish a diagnosis. A follow-up to this program is the use of correlation techniques to automatically "recognize" ECG patterns and classify them. It is proposed that the general pattern recognition problem associated with the electrocardiogram be solved by what is commonly referred to as the matched filter technique. This concept shows promise for additional development as it operates on a much greater amount of information than the standard clinical parameters.

The matched filters technique is a relatively new and important aspect to the increasingly large study of information theory. This concept assumes the specific notion that the correlation of one waveform with another can be

carried out by passing the first waveform through a linear system whose impulse response is the time reverse of the second waveform, and then observing the output at a certain instant of time. If the two waveforms are made the same we say that the filter is "matched" to the input waveform. The filter output as a function of real time is then the autocorrelation function of the waveform as will be concluded from that which follows.

If $s(t)$ is any physical waveform having a Fourier transform, then a filter which is matched to $s(t)$ is, by definition, one with impulse response

$$h(\tau) = ks(\Delta - \tau) , \quad (1)$$

where k and Δ are arbitrary constants. The choice of Δ must be compatible with physical realizability. In order to envisage the form of $h(\tau)$, consider Figure 2, in part (a) of which is shown a wave train, $s(t)$, lasting from t_1 to t_2 . By reversing the direction of time in part (a), i.e., letting $\tau = -t$, one obtains the reversed train, $s(-\tau)$, of part (b). If this latter waveform is now delayed by Δ seconds, and its amplitude multiplied by k , the resulting waveform - part (c) of Figure 2 - is the matched filter impulse response of 1.

The transfer function of a matched filter, which is the Fourier transform of the impulse response, has the form

$$\begin{aligned} H(j2\pi f) &= \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau \\ &= k \int_{-\infty}^{\infty} s(\Delta - \tau) e^{-j2\pi f\tau} d\tau \end{aligned}$$

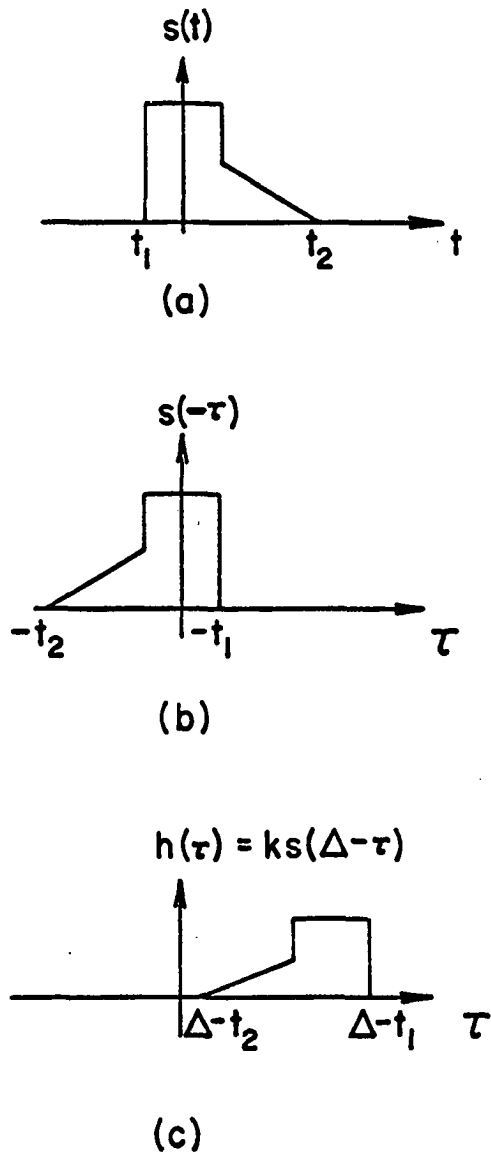


Figure 2. Pertaining to the definition of a matched filter

- (a) A wave train
- (b) The reversed train
- (c) A matched filter impulse response

$$= k e^{-j\pi f \Delta} \int_{-\infty}^{\infty} s(\tau') e^{j2\pi f \tau'} d\tau' , \quad (2)$$

where the substitution $\tau' = \Delta - \tau$ has been made in going from the third to the fourth member of 2. Now the spectrum of $s(t)$, i.e., its Fourier transform, is

$$S(j2\pi f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt . \quad (3)$$

It should be noted that this is a density spectrum. If $s(t)$ is, e.g., a voltage waveform, $S(j2\pi f)$ is a voltage density, and its integral from f_1 to f_2 (plus that from $-f_2$ to $-f_1$) is the part of the voltage in $s(t)$ originating in the band of frequencies from f_1 to f_2 . Comparison of 2 and 3 reveals, then, that

$$\begin{aligned} H(j2\pi f) &= k S(-j2\pi f) e^{-j2\pi f \Delta} \\ &= k S^*(j2\pi f) e^{-j2\pi f \Delta} . \end{aligned} \quad (4)$$

That is, except for a possible amplitude and delay factor of the form $ke^{-j2\pi f \Delta}$, the transfer function of a matched filter is the complex conjugate of the spectrum of the signal to which it is matched.

In order to appreciate the significance of the matched filter development, consider the following. A rectangular pulse shown in Figure 3(a) of amplitude 1 and duration from $t = -\frac{T_1}{2}$ to $t = +\frac{T_1}{2}$.

The Fourier transform of $s(t)$ is obtained as

$$S(j2\pi f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$$

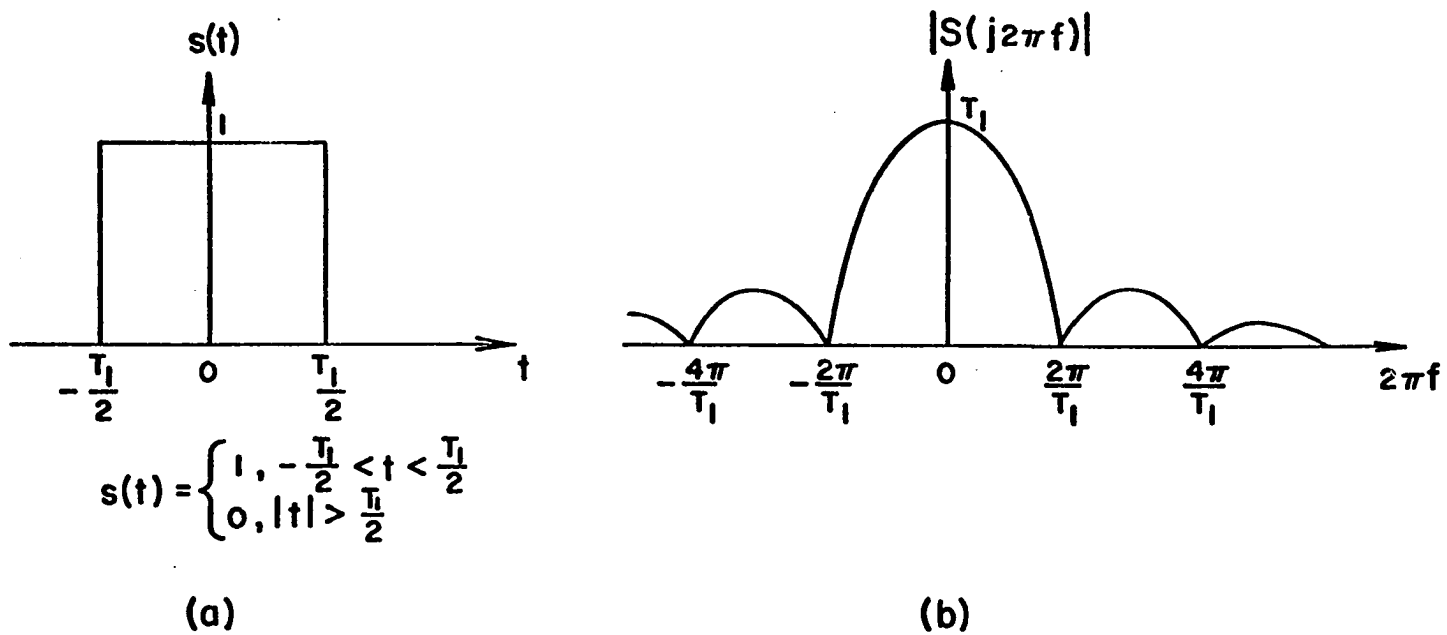


Figure 3. Pertaining to the development of a matched filter transfer function

(a) Rectangular pulse

(b) The Fourier transform of the rectangular pulse

$$\begin{aligned}
&= \frac{T_1}{2} \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} e^{-j2\pi ft} dt \\
&= 2 \int_0^{\frac{T_1}{2}} \cos 2\pi ft dt \\
&= T_1 \frac{\sin \frac{2\pi f T_1}{2}}{\frac{2\pi f T_1}{2}} \quad (5)
\end{aligned}$$

The magnitude of $S(j2\pi f)$ is sketched as a function of frequency in Figure 3(b). The transfer function of the matched filter in this example is obtained from $H(j2\pi f) = k S^*(j2\pi f) e^{-j2\pi f \Delta}$. Now $S^*(j2\pi f)$ is obtained as

$$\begin{aligned}
S^*(j2\pi f) &= \int_{-\infty}^{\infty} s(t) e^{+j2\pi ft} dt \\
&= \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} e^{+j2\pi ft} dt \\
&= 2 \int_0^{\frac{T_1}{2}} \cos 2\pi ft dt \\
&= T_1 \frac{\sin \frac{2\pi f T_1}{2}}{\frac{2\pi f T_1}{2}} \quad (6)
\end{aligned}$$

and therefore the matched filter transfer function is

$$H(j2\pi f) = k T_1 \frac{\sin \frac{\omega T_1}{2}}{\frac{\omega T_1}{2}} e^{-j2\pi f \Delta} \quad (7)$$

For the case where $k = 1$ and $\Delta = \frac{T_1}{2}$, the transfer function of the matched filter is equal to the spectrum of the signal to which it is matched with a phase shift of $e^{-j2\pi f \frac{T_1}{2}}$. Figure 4(a) shows the magnitude of $H(j2\pi f)$ as a function of frequency and Figure 4(b) indicates the associated phase shift.

Consider then, the system of Figure 5. A signal, $s(t)$, say of duration T , may be imagined to be generated by exciting a filter, whose impulse response is $s(\tau)$, with a unit impulse at time $t = 0$. To this signal is added a white noise waveform, $n(t)$. The sum signal, $x(t)$, is then passed into a filter, matched to $s(t)$, whose output is denoted by $y(t)$. The output signal may be resolved into two components,

$$y(t) = y_s(t) + y_n(t) \quad (8)$$

the first of which is due to $s(t)$ alone, and the second to $n(t)$ alone.

The response to an input $s(t)$ alone, i.e., an input not contaminated by white noise, of a linear filter with impulse response $h(\tau)$ is

$$y_s(t) = \int_{-\infty}^{\infty} h(\tau) s(t - \tau) d\tau \quad (9)$$

If $h(\tau) = s(\Delta - \tau)$, then

$$y_s(t) = \int_{-\infty}^{\infty} s(\Delta - \tau) s(t - \tau) d\tau \quad (10)$$

Making first the substitution $\tau' = \Delta - \tau$, and then the substitution

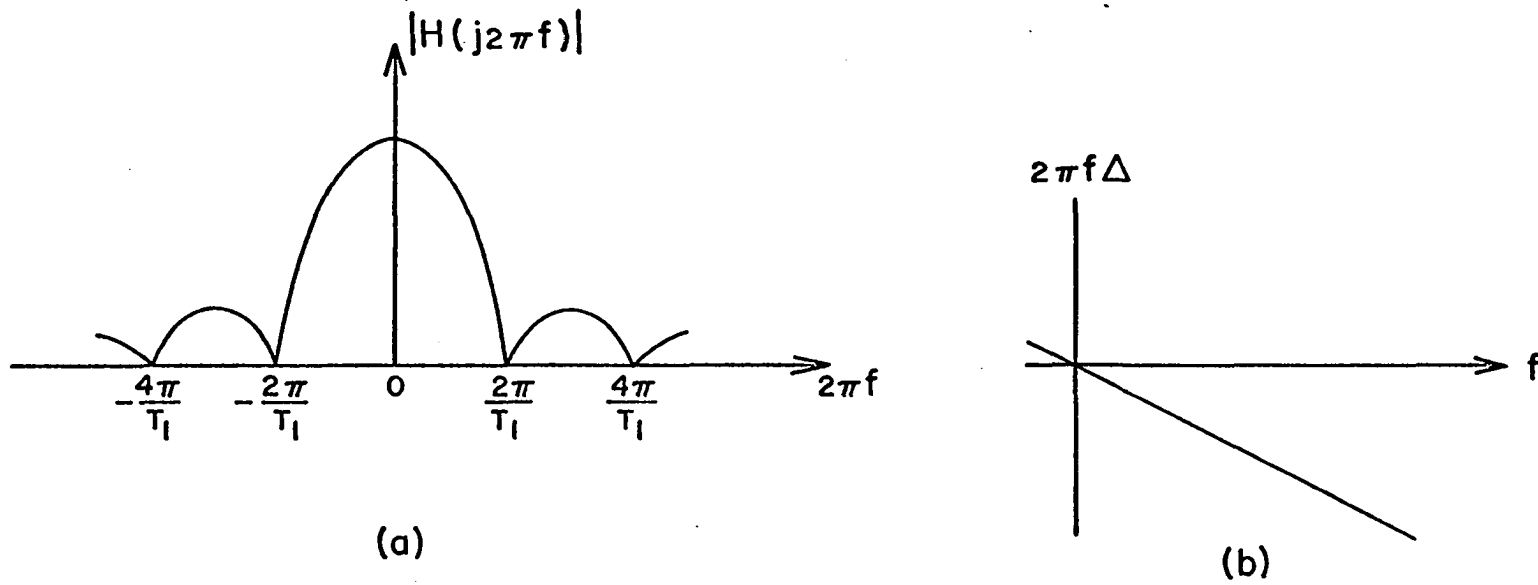


Figure 4. A matched filter transfer function

(a) Magnitude of the matched filter transfer function $H(j2\pi f)$ for the rectangular pulse

(b) The associated phase shift

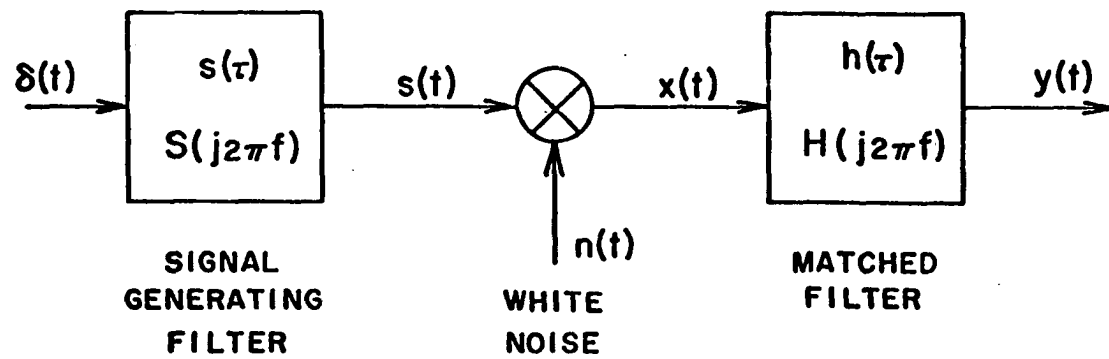


Figure 5. Illustration of matched filter properties

$t' = t - \Delta$ obtains

$$y_s(t') = \int_{-\infty}^{\infty} s(\tau') s(t' + \tau') d\tau' \quad (11)$$

which, when considered in finite form over a period T of a single pulse becomes

$$y_s(t' + \Delta) = \frac{1}{2T} \int_{-T}^T s(\tau') s(t' + \tau') d\tau' \quad . \quad (12)$$

It is noted that this form is exactly the same as the form of the finite autocorrelation function.

If the input to the filter of Figure 5 was contaminated by the white noise component, $n(t)$, then the response of a linear filter with impulse response $h(\tau) = s(\Delta - \tau)$ would be something other than that previously obtained. The filter would not be exactly matched to $x(t)$, the sum signal of $s(t) + n(t)$. Thus, the response of the matched filter becomes, when considered in finite form over a period T of a single pulse,

$$y(t) = \frac{1}{2T} \int_{-T}^T s(\tau) x(t + \tau) d\tau \quad . \quad (13)$$

This may be recognized as the form taken by the cross correlation function between the input signal and the desired output.

It becomes apparent from the preceding development that the filter output as a function of real time for a "pattern match" is then the autocorrelation function of the waveform. The autocorrelation function of a random time stationary function $s(t)$ is defined as

$$\phi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T s(t) s(t + \tau) dt \quad . \quad (14)$$

It is a measure of how the function $s(t)$ is correlated with itself after an elapsed time. The variance of $s(t)$ is given by

$$\overline{s^2(t)} = \sigma^2 = \phi(0) \quad (15)$$

and $\phi(\tau)$ never exceeds $\phi(0)$. Thus, for a function $s(t)$ defined by $s(t) = \sin \omega t$, which is the waveform as shown in Figure 6(a), the autocorrelation function can be computed by shifting $s(t)$ to the left by an amount τ and averaging the product of the shifted function and the original function. The value of the autocorrelation function may be computed as follows.

$$\begin{aligned} \phi(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \sin \omega t \sin (\omega t + \tau) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \sin \omega t \sin(\omega t + \tau) dt \end{aligned} \quad (16)$$

Since $s(t)$ is periodic, integrate over one period to obtain

$$\begin{aligned} \phi(\tau) &= \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t \cos \omega \tau dt \\ &= \frac{1}{2} \cos \omega \tau . \end{aligned} \quad (17)$$

A sketch of the resultant autocorrelation function is as shown in Figure 6(b).

The previous discussion has demonstrated that the matched filter output as a function of real time can be described in terms of the correlation functions of the waveform. If the minimization of the mean-square error is adopted as the design criterion, then it is known that the signals are adequately described by the correlation functions.

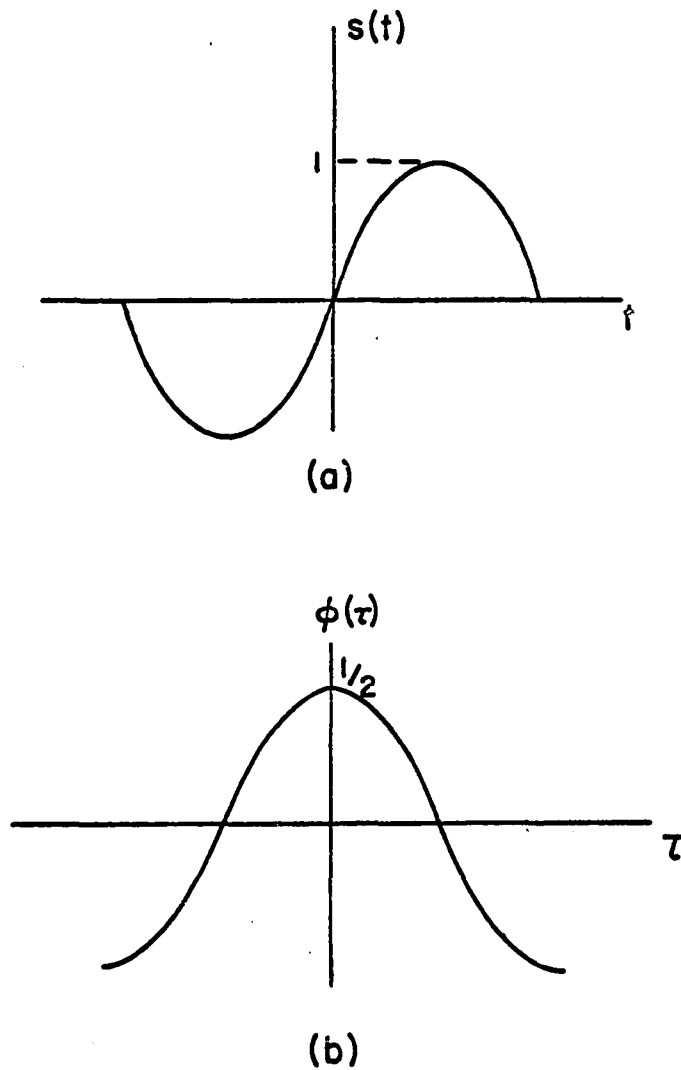


Figure 6. Pertaining to the autocorrelation function

(a) The given function $s(t) = \sin \omega t$

(b) The autocorrelation function of $s(t)$

Consider the context in which the matched filter first made its appearance as depicted in Figure 7. Suppose a waveform, $x(t)$, has been received, which consists either solely of a white noise, $n(t)$, or of $n(t)$ plus a signal $s(t)$ of known form. It is wished to determine which of these contingencies is true by operating on $x(t)$ with a linear filter in such a way that if $s(t)$ is present, the filter output at some time $t = \Delta$ will be considerably greater than if $s(t)$ is absent. Now, since the filter has been assumed to be linear, its output, $y(t)$, will be composed of a noise component $y_n(t)$, due to $n(t)$ only, and in addition, if $s(t)$ is present, a signal component $y_s(t)$, due to $s(t)$ only. According to Turin (19), when the filter of Figure 7 is matched to $s(t)$, a maximum value of the signal power to noise power is obtained at $y(\Delta)$. The matched filter response indicated diagrammatically in Figure 7 illustrates the above comments.

Application of the matched filter concept to the pattern recognition problem of electrocardiography utilizing a data processing system is the ultimate objective of this dissertation. A consideration of the preceding comments of this section indicates that a digital matched filter system would be comprised of $s_1(t)$, $s_2(t)$, \dots $s_m(t)$ standard patterns. These standard patterns would be stored in memory within a computer and then a "recognition" scheme could be developed by a direct method of comparing unknown ECG waveforms with known standards. The "recognition" process would involve literally following the edicts of the correlation function

$$y(t_0) = \int_I s(t - t_0) x(t) dt \quad . \quad (18)$$

That is, one could multiply the incoming unknown waveform, $x(t)$, by a

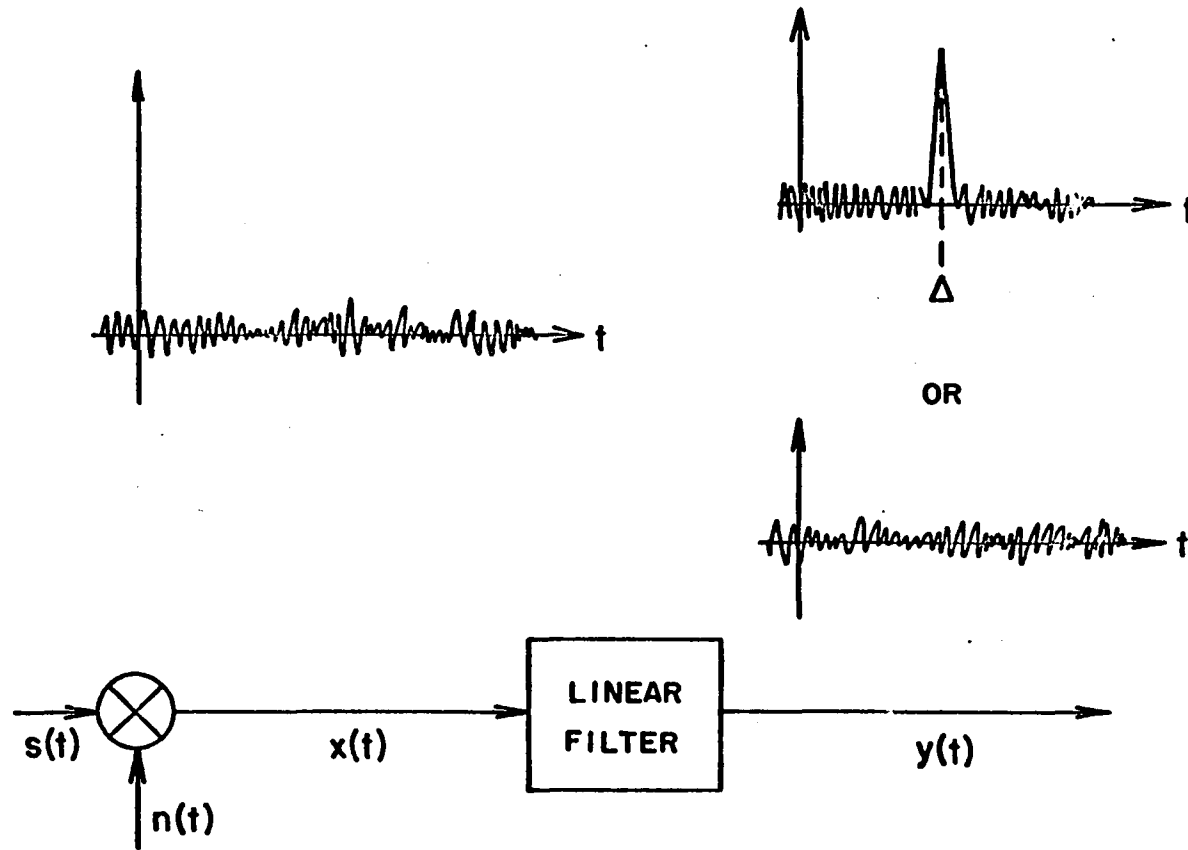


Figure 7. Pertaining to the maximum signal-to-noise ratio

stored replica of a standard signal waveform, $s(t)$, delayed by t_0 . The product, integrated over the observation interval, is $y(t_0)$, and is compared with a threshold value λ . This simple detection scheme is depicted in Figure 3.

Summarizing briefly the comments and observations of this section, it is apparent that a matched filter detection scheme holds promise for obtaining a machine diagnosis of the clinical electrocardiogram. It is to be noted, however, that pertinent characteristics of the correlation functions must be considered in the experimental analysis. Also, the nature and form of the ECG waveform must be carefully examined to obtain logical results. In applying the matched filter concept to the electrocardiogram, these factors will be elaborated upon in the discussion of the experimental procedure which follows.

B. Experimental Procedure

It has been indicated in the preceding section that an application of correlation techniques to the ECG waveform might enable one to "recognize" a cardiac disease state. If this is possible, then, through the utilization of a digital computer to accomplish the computations required, an important tool has been obtained to aid the medical doctor. Needless to say, a machine medical diagnosis of the electrocardiogram is an exciting consideration. Before pursuing these thoughts further, it is necessary to investigate the applicability of correlation techniques to the waveform of the electrocardiogram.

The characteristics of the electrocardiographic waveform can be enumerated briefly. It is an approximately periodic cycle composed of

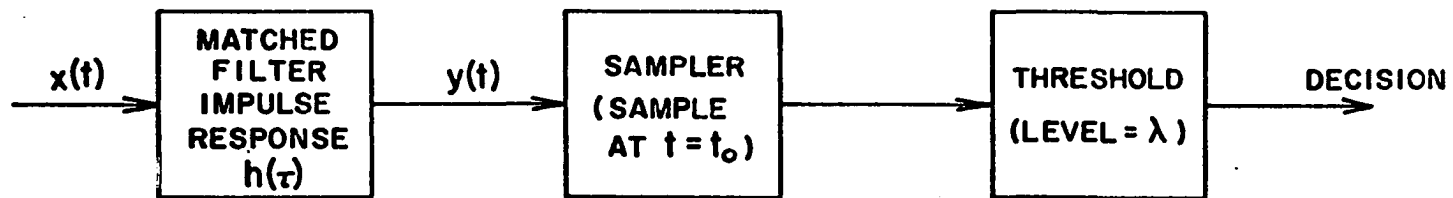


Figure 3. A simple correlation detector

several pulses. This waveform possesses certain distinctive features, which are always present, regardless of the patient being recorded. The heart rate varies from patient to patient and thus the length of the cardiac waveform is variable with patient. It is also known that the length of the waveform is not directly proportional to the heart rate.

Certain properties, pertinent to this study, of the autocorrelation and cross-correlation functions should be noted. The value of the autocorrelation function never exceeds the value for zero argument, that is, $|\phi_{11}(\tau)| \leq \phi_{11}(0)$. A given autocorrelation function may correspond to any number of time functions, however any given time function has only a single autocorrelation function. A given cross-correlation function does not necessarily possess a maximum at $\tau = 0$, that is, $|\phi_{12}(\tau)| \neq \phi_{12}(0)$.

In the application of the principles discussed in the preceding section, a perfect pattern match insures that an autocorrelation function is obtained with the result that $\phi_{11}(\tau)$ is maximum at $\tau = 0$. It is anticipated that this particular property of the correlation function will be extensively utilized in this experimental investigation. A perfect pattern match is not, in general, to be expected and therefore one obtains the cross-correlation function which means that $\phi_{12}(\tau)$ is not necessarily maximum at $\tau = 0$. This may be viewed in the context that the input to the digital filter is contaminated by noise and, thus, an identical pattern match would not be obtained. If however a normalization procedure is performed, both in amplitude and time, on each test signal or pattern, such that each test signal is "matched" to the standard pattern, then significant results may be anticipated. More comments concerning this normalization procedure will be elaborated shortly. Let it be noted at

this point however, that logical results can be obtained, via correlation techniques, only if the nature and form of typical waveforms associated with the various cardiac disease states are carefully examined.

From the above comments it becomes apparent that the experimental procedure will involve extensive use of the cross-correlation function, which reduces to the autocorrelation function for a pattern match. The periodic nature of the ECG waveform simplifies the correlation functions. If the period of the heart rate is T , the finite cross-correlation function is

$$\phi_{sx}(\tau) = \frac{1}{2T} \int_{-T}^T S_m(t) X_n(t + \tau) dt \quad (19)$$

If sequential samples are to be dealt with, as in the case of machine computation, and a shift in sample number defines a shift in time, the cross-correlation function of a sequence of N numbers may be written as

$$\phi_{sx}(j) = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{k=-N}^N S_k X_{k+j} \quad (20)$$

This represents the correlation function for the discrete case and is useful for the sampled data technique applicable in this study. Since the wave is periodic it is necessary to perform correlation only over a single period.

As mentioned previously in this discussion it was suggested that an approximate pattern match might be obtained by normalization of each individual wave. All of the electrocardiographic cycles which were to be compared with one another were initially made the same length in time as that of the selected reference or standard patterns. The lengths of the cycles were increased or decreased in length by requiring that each waveform

be sampled the same number of times. In this investigation the standard patterns were selected as near as possible to coincide with average normal heart rate. Initially bearing this in mind, a standard pattern for a normal cardiac cycle was selected with a heart rate of 75 beats per minute. Amplitude normalization was realized by requiring that the integral of the area squared for the standard patterns be set equal to unity and that all other waveforms be adjusted in amplitude to meet this specification. This criteria insures that for an optimum pattern match a numerical value of unity can be effected, but realistically something less than unity will be obtained when the test patterns are compared to the standard patterns. Appropriate details of the normalization techniques will be discussed more completely in succeeding paragraphs.

Shown in Figure 9 is the waveform of a normal electrocardiogram. This typical waveform was recorded by one lead, namely V_6 , for a patient at Iowa Methodist Hospital. The waveform has been expanded in amplitude by a factor of ten and in time by a factor of five in order to obtain data in usable digital form. After consultation with a cardiologist it was determined that this particular waveform was representative of a normal ECG and would be acceptable as a reference or standard pattern. This standard pattern has been labeled $s_1(t)$ for convenience. As can be seen from Figure 9 the duration of the pattern is 0.8 seconds. Sampling this particular cycle every 0.01 second for a total of 81 sample points corresponds to a sampling rate of 100 samples per second. Mere chance alone did not determine this sampling rate. Two factors had to be considered. One factor of some concern was that the sampling had to be performed

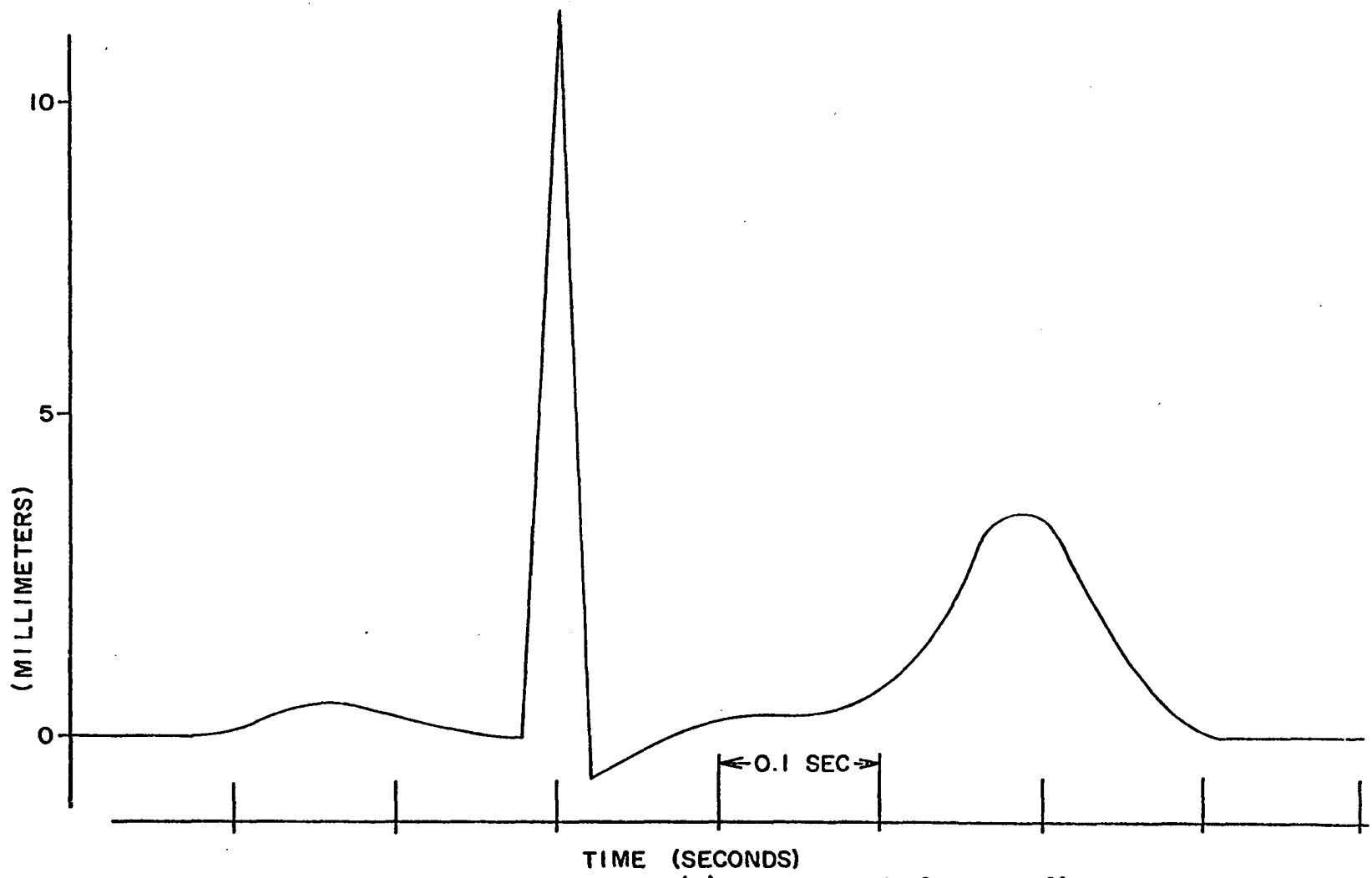


Figure 9. Standard pattern $s_1(t)$ for a normal electrocardiogram

manually for a large number of cases which established a certain practical physical limitation. More important, however, was a fundamental concept known from the study of information theory. This may be stated as follows: if the approximate duration of a known signal is T seconds and if W is its approximate spectral bandwidth and if $2TW \gg 1$, then it is known that the function is determined everywhere to a high degree of accuracy by its values at $2TW$ sampling points spaced at time locations $1/2W$ apart. From studies carried out by other researchers it is known that approximately 50 cycles per second is the upper limit of the frequency content of the electrocardiogram, therefore the sampling points should be spaced at $\frac{1}{2 \times 50} = 0.01$ seconds apart for this cardiac cycle. Researchers at the National Institute of Health have indicated that, for all practical purposes, about 30 cycles per second is reasonable for the upper limit of frequency. Thus for a heart rate (H.R.) of 75 beats per minute the cycle length (C.L.) in seconds is obtained from

$$\begin{aligned} \text{C.L.} &= \frac{1}{\text{H.R.} \frac{\text{BEATS}}{\text{MIN}}} \times 60 \frac{\text{SEC}}{\text{MIN}} \\ &= \frac{1}{75} \times 60 = 0.8 \text{ seconds.} \end{aligned} \quad (21)$$

If 80 samples are taken over this cycle length, then this corresponds to one sample for each 0.01 second, or 100 samples per second. Counting both the end points of the waveform, one obtains a total of 81 samples for the complete cycle.

As stated previously, all the cardiac cycles to be used were made the same length in time. This was accomplished by taking 81 sample points over the complete cycle for each signal considered. What is implied from the

preceding comment, then, is that the sampling rate was varied with cycle length. Considering this in more detail, it is noted that

$$\frac{C.L.}{80} = t \text{ second/sample}, \quad (22)$$

or that the sampling rate (S.R.) is given by

$$S.R. = \frac{1}{t} = \frac{80 \text{ sample}}{C.L. \text{ second}}. \quad (23)$$

It is noted further that there is a linear relationship between sampling rate and heart rate, since

$$S.R. = \frac{80}{C.L.} = 80 \times \frac{H.R.}{60} = \frac{4}{3} H.R. \quad (24)$$

This relationship is shown graphically in Figure 10. Figure 11 depicts in graphical form the relationship between the cycle length and heart rate. One further observation should be made at this point, i.e., the sampled data obtained conformed to the requirements of the sampling theorem in the time domain throughout the range of cycle lengths or heart rates of interest.

It is logical to inquire as to the feasibility of the time normalization discussed in the preceding paragraphs. Earlier in the discussion of this section, it was noted that the length of an electrocardiographic waveform is not proportional to heart rate. Medical data has been published, however, which indicates a functional relationship over a considerable portion of the cardiac cycle between interval lengths and heart rate. Of particular interest to this dissertation was the data available for the Q-T interval. This particular interval comprises the major segments of interest in the application of correlation techniques to the ECG waveform. The Q-T interval is an interval measured from the beginning of the Q wave

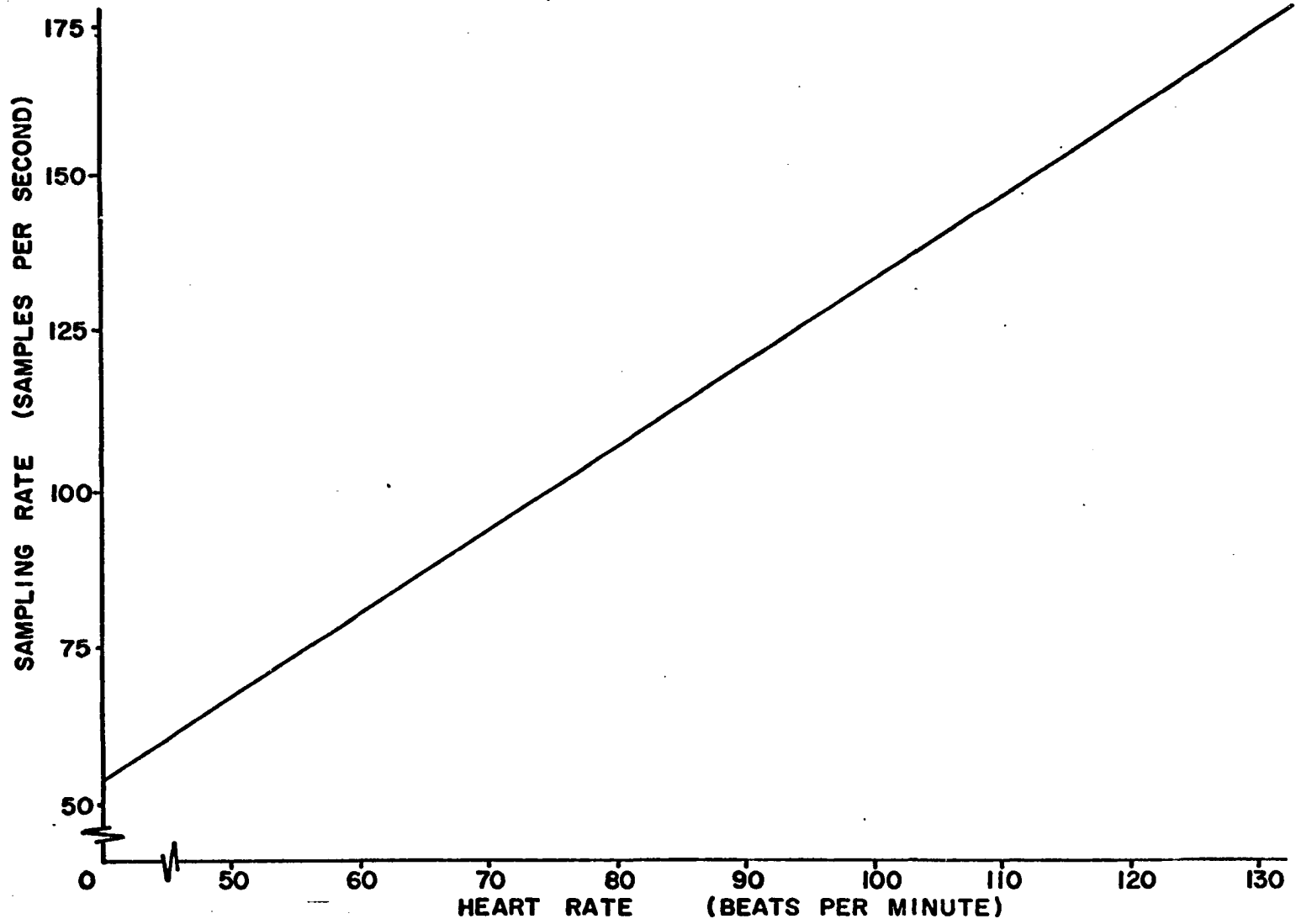


Figure 10. Sampling rate as a function of heart rate

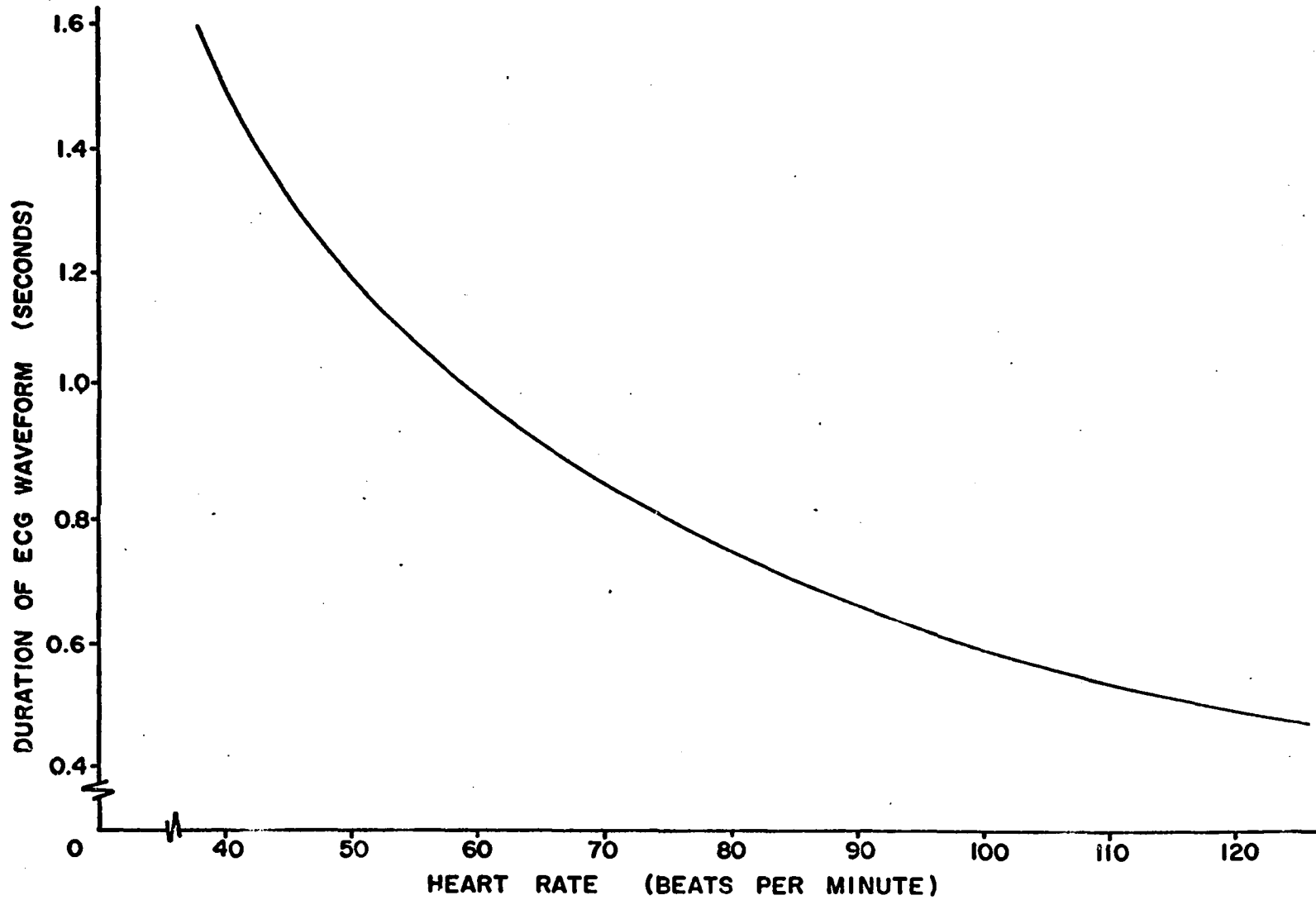


Figure 11. Cycle length as a function of heart rate

to the end of the T wave. In Figure 12 is shown a curve of this interval as related to heart rate.

As a first approximation, it was assumed that the Q-T interval is proportional to the total cycle length. In order to test the validity of this assumption, the ratio of Q-T to cycle length was plotted as a function of heart rate. This is shown in Figure 13. A casual glance at this particular curve reveals that the ratio of Q-T interval to cycle length is not constant with respect to heart rate. It does not, however, exhibit a radical variation over the entire heart rate range. One could reasonably expect to find a suitable heart rate range over which, to a good approximation, the time normalization method utilized in this study may be valid. By application of a correction factor, it would appear reasonable to anticipate an improved time normalization. If the standard pattern were selected with a heart rate of 75 beats minute, then a curve of the ratio $Q-T/Q-T @75$ plotted against heart rate would be applicable for a correction on time normalization. Such a curve has been constructed and is as shown in Figure 14. As is indicated by this curve, there is a heart rate range from 60 to 100 over which the time normalization, on the average, will be correct to within ten percent of the norm. As will be discussed later, this deviation appears to be acceptable. For a heart rate of 50 beats per minute, for example, it may be expected that a correction in normalization is required. At this particular heart rate it is observed from the curve of Figure 14 that the Q-T time at 50 beats per minute is 1.18 greater than that of the Q-T reference at 75 beats per minute. This curve, then, indicates the change in displacement, as function of time, for the essential segments of interest. Thus, for this example, a total of 1.18×81 sample

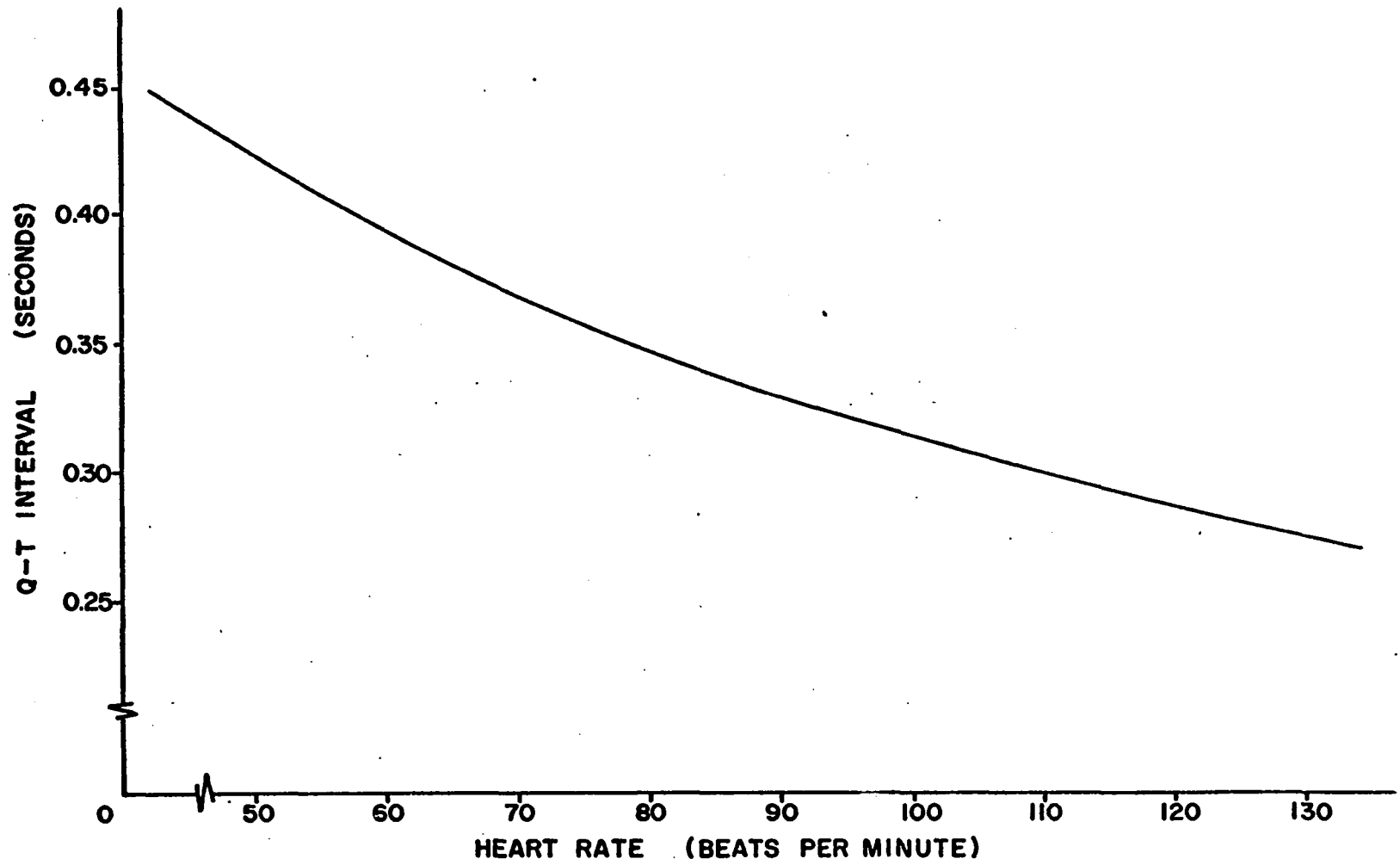


Figure 12. Q-T interval as a function of heart rate

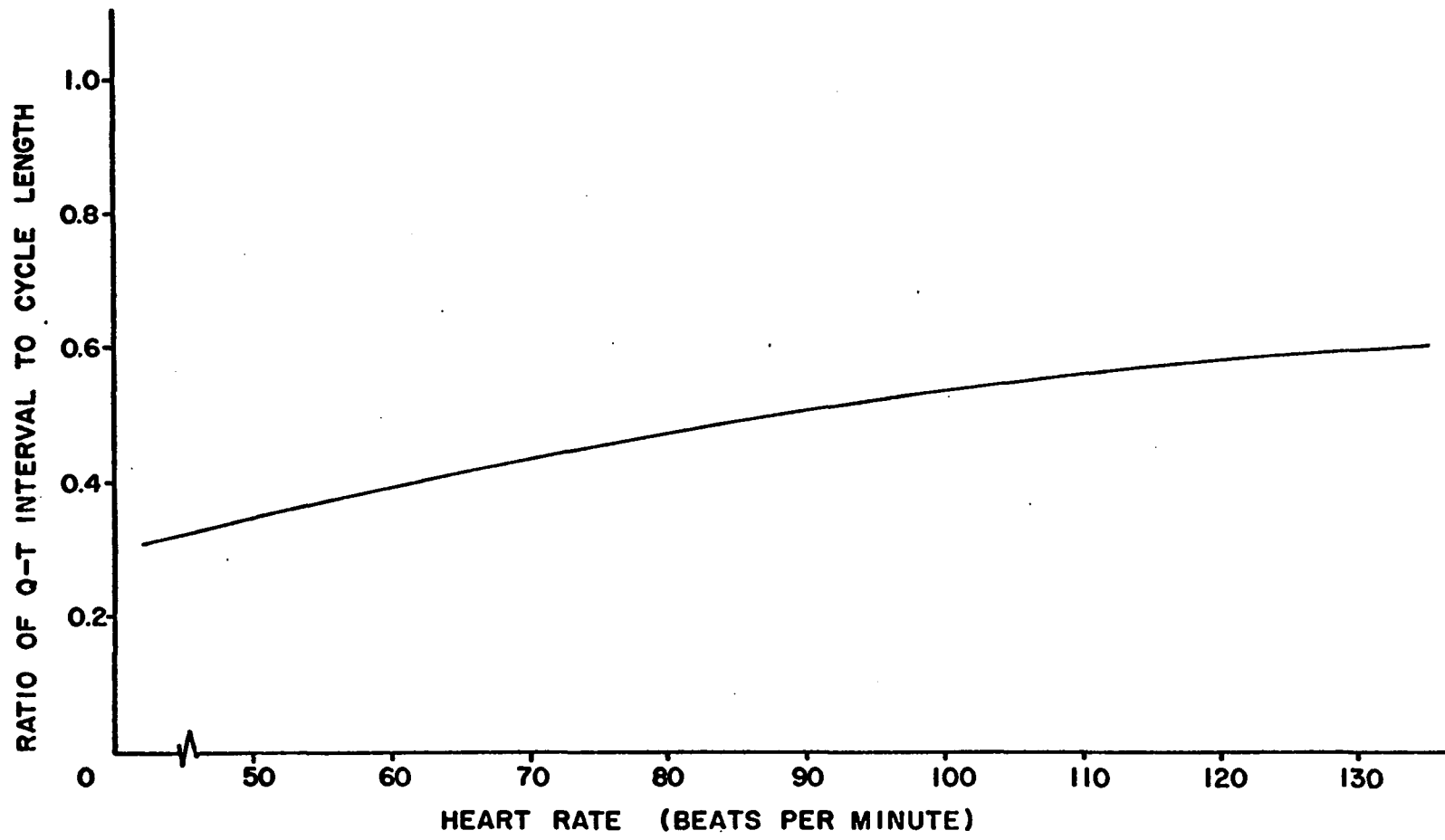
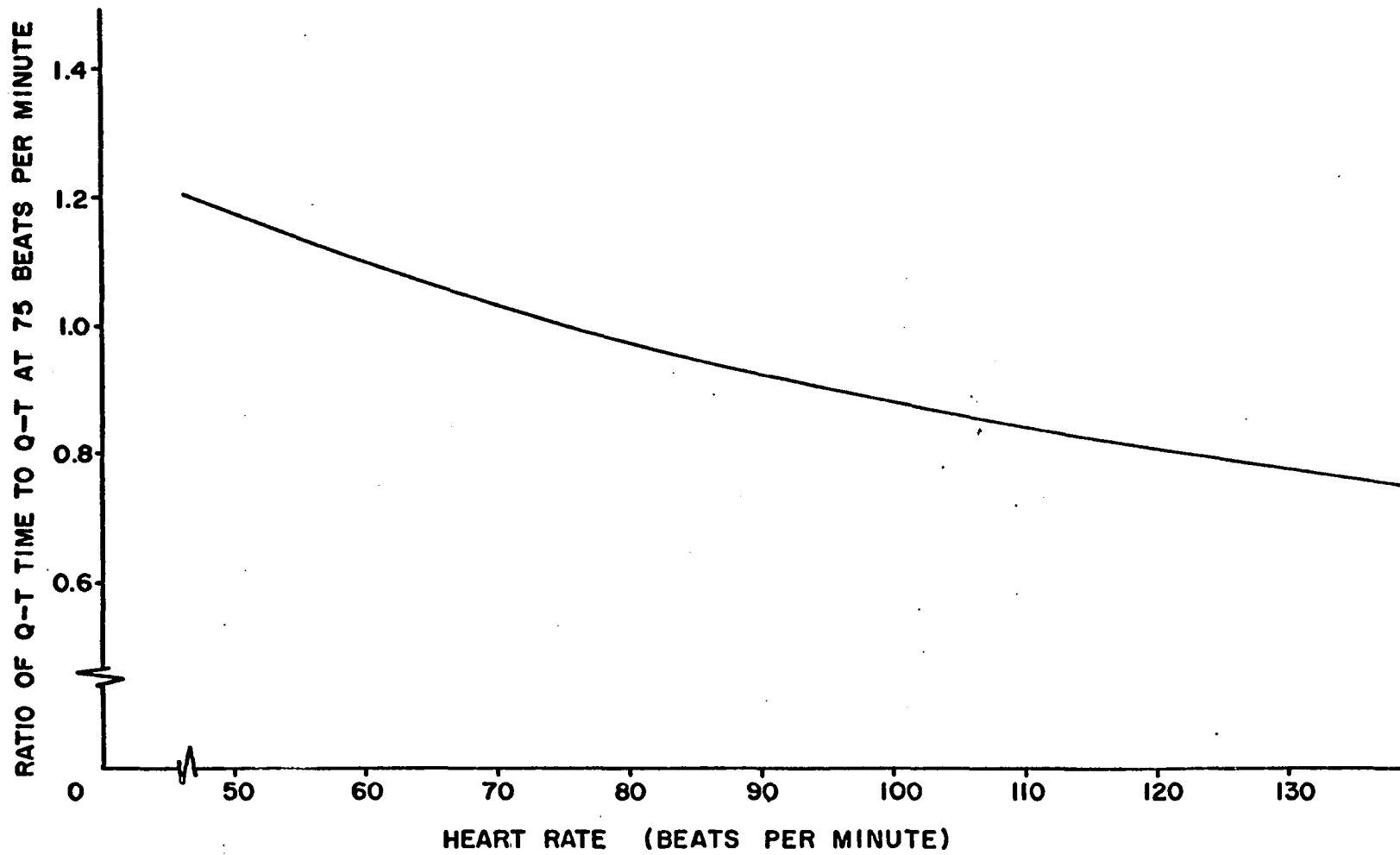


Figure 13. Ratio Q-T interval to cycle length as a function of heart rate



04

Figure 14. Per unit Q-T time as a function of heart rate

points, i.e., 96 sample points are required over the cycle length.

Much has been said in the preceding discussion about time normalization. Of equal importance, however, was the consideration of amplitude normalization. After all waveforms had been reduced to the same length in time, the amplitudes were modified by adopting the integral of the area squared criteria mentioned previously. The logic of this criteria becomes evident from an examination of the correlation functions. In correlating mathematically one function with another, the average of the area of a curve is obtained, i.e., the product of two time functions is integrated and then averaged over the interval of interest. This criteria, then, is based on the fundamentals of the mathematics of correlation.

Elaborating on the above paragraph seems appropriate. Assume that a standard pattern $s_1(t)$ has been selected, then a function may be calculated as

$$\int_0^T [s_1(t)]^2 = K . \quad (25)$$

Note that the data for all waveforms is specified by 61 data points and therefore an averaging of this function is not necessary. A modified autocorrelation, i.e., a perfect fit pattern match, computation for zero delay is thus the result. After K has been calculated then an unknown waveform or signal $x_n(t)$ may be normalized with respect to the reference pattern by requiring that

$$\int_0^T [\alpha x_n(t)]^2 = K , \quad (26)$$

where α is the correction factor by which $x_n(t)$ is modified. This process

may be easily carried out by direct computation on a digital computer. It was specified for the machine calculation of this integral that it be obtained to be at least equal to $0.99K$. Amplitude normalization, as performed through the use of the above method, was carried out in a straightforward manner in this study.

After the patterns have been fitted according to both time and amplitude, a "recognition" of test patterns can be effected. This was accomplished by evaluating the integral

$$\int_0^T [s_1(t)][\alpha x_n(t)]dt \quad . \quad (27)$$

This calculation will obtain a constant K' which is always less than and at most equal to K . On a per unit basis, the result of dividing K' by K establishes a "recognition" scale of values ranging from 0 through 1.

In brief summary, this section has outlined the various aspects of applying correlation techniques to the electrocardiographic waveform. Pertinent characteristics of the correlation functions as well as that of the cardiac waveform have been examined in some detail. The normalization techniques discussed have entailed, basically, a concept of preserving, to a considerable extent, the general shape of each individual pattern. Experimental results, indicating the success of these methods will be discussed in the ensuing section.

C. Experimental Results

The experimental work entailed a direct application of the notions presented in the two preceding sections. Records were secured of patients who had undergone a clinical electrocardiogram at Iowa Methodist Hospital

of Des Moines. Shown in Figure 15 and Figure 16 is a facsimile of the typical record used for physician examination at Iowa Methodist. Aside from the regular patient identification information, Figure 15 displays the twelve waveforms routinely recorded. Figure 16 indicates the pertinent measurements that were made and the interpretation or diagnosis made by the examining physician. Fifty-five such records were utilized in the experimental effort, but only one waveform, from the usual twelve recorded, was necessary in this attempt at a machine medical diagnosis. Lead V_6 , shown in the lower right hand corner of Figure 15, obtained the waveform required from all electrocardiograph reports used in this study. A medical interpretation had been provided on all these records by one of five medical doctors associated with Iowa Methodist.

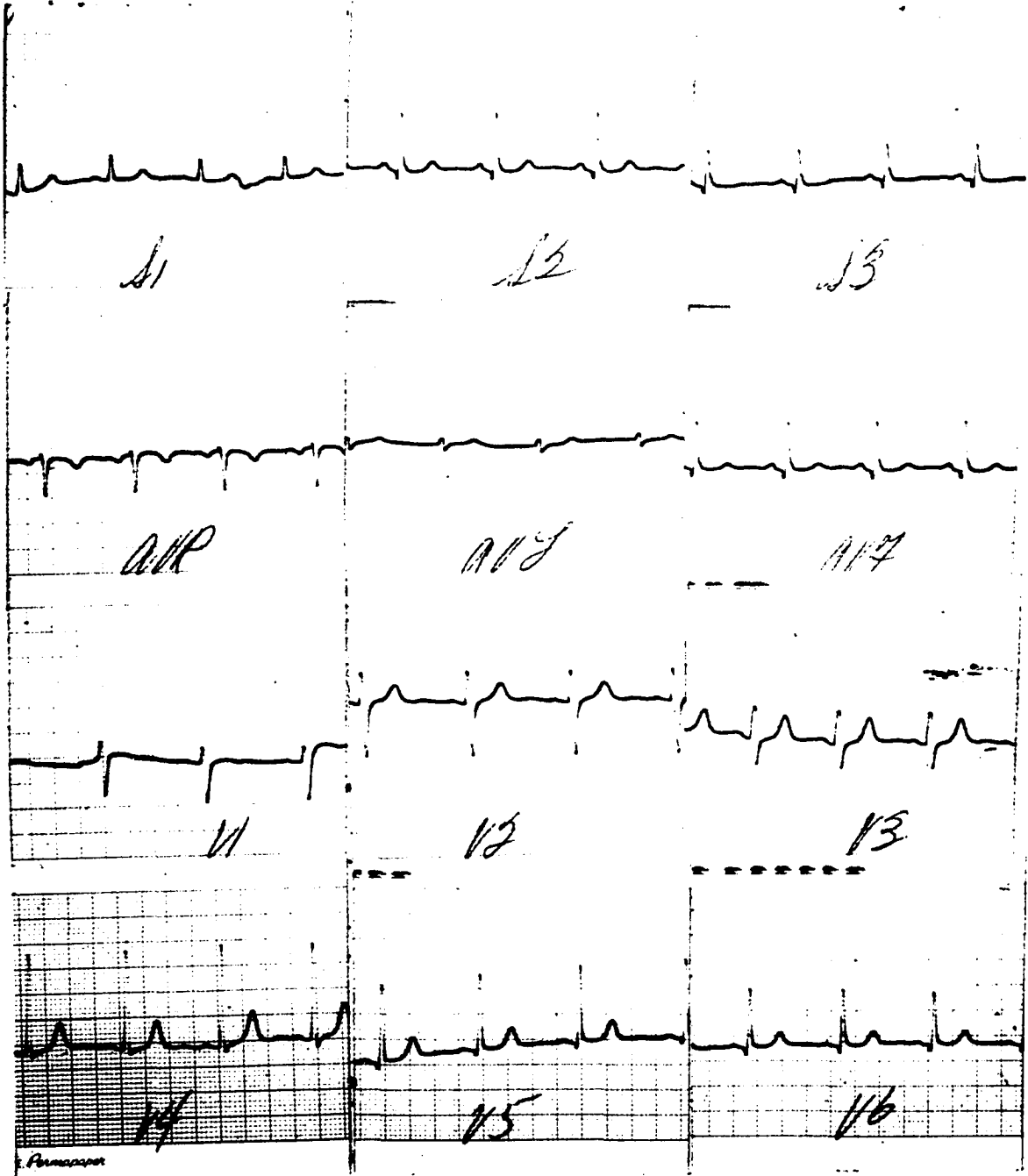
Associated with a relatively large group of clinical electrocardiograms one would expect to obtain a variety of cardiac disease states. This was indeed the situation with respect to the records secured for this study. Of the fifty-five cases, thirty-five of these had been initially interpreted as being normal, while twenty were initially interpreted as being abnormal. As the machine study progressed, one abnormal indication was changed to a normal interpretation after consultation with a cardiologist at Iowa Methodist. Thus, there were a total of thirty-six normal patterns and nineteen abnormal patterns that were useful for this dissertation. This does not imply, however, that each and every pattern was distinct in and of itself. Many waveforms had similar characteristics, this would be expected. In total, then, there were seven relatively distinct patterns which appear to be representative of the various cardiac disease states. These are indicated on the succeeding pages by Figures 17 through 23.

Figure 15. Electrocardiograph report

IOWA METHODIST HOSPITAL
RAYMOND BLANK MEMORIAL HOSPITAL FOR CHILDREN

Patient: MURPHY, LILLIAN C. Age: 2 Room No. 304

Date Taken: 3-5-62 Dr. _____ Hosp. No. 1-15



Permapaper

MR-17

ELECTROCARDIOGRAPH REPORT

Figure 16. Interpretation of electrocardiograph

RATE: 75. RHYTHM: Normal sinus.

P WAVES: Normal.

PR SEC: .15. QRS: Conduction time .06 seconds; normal axis. There is a Q wave in leads II, III, aVF, V5, and V6.

SV-1: RV-6: RV-1: SV-6:

ST SEG: No significant abnormalities noted.

T WAVES: Within normal limits throughout the tracing.

OTHER:

INTERPRETATION:

There are no definitely abnormal findings in this tracing.

DICTATED BY _____ M. D.
H. G. BARK

HOB:cva
3-7-62
742-B-2

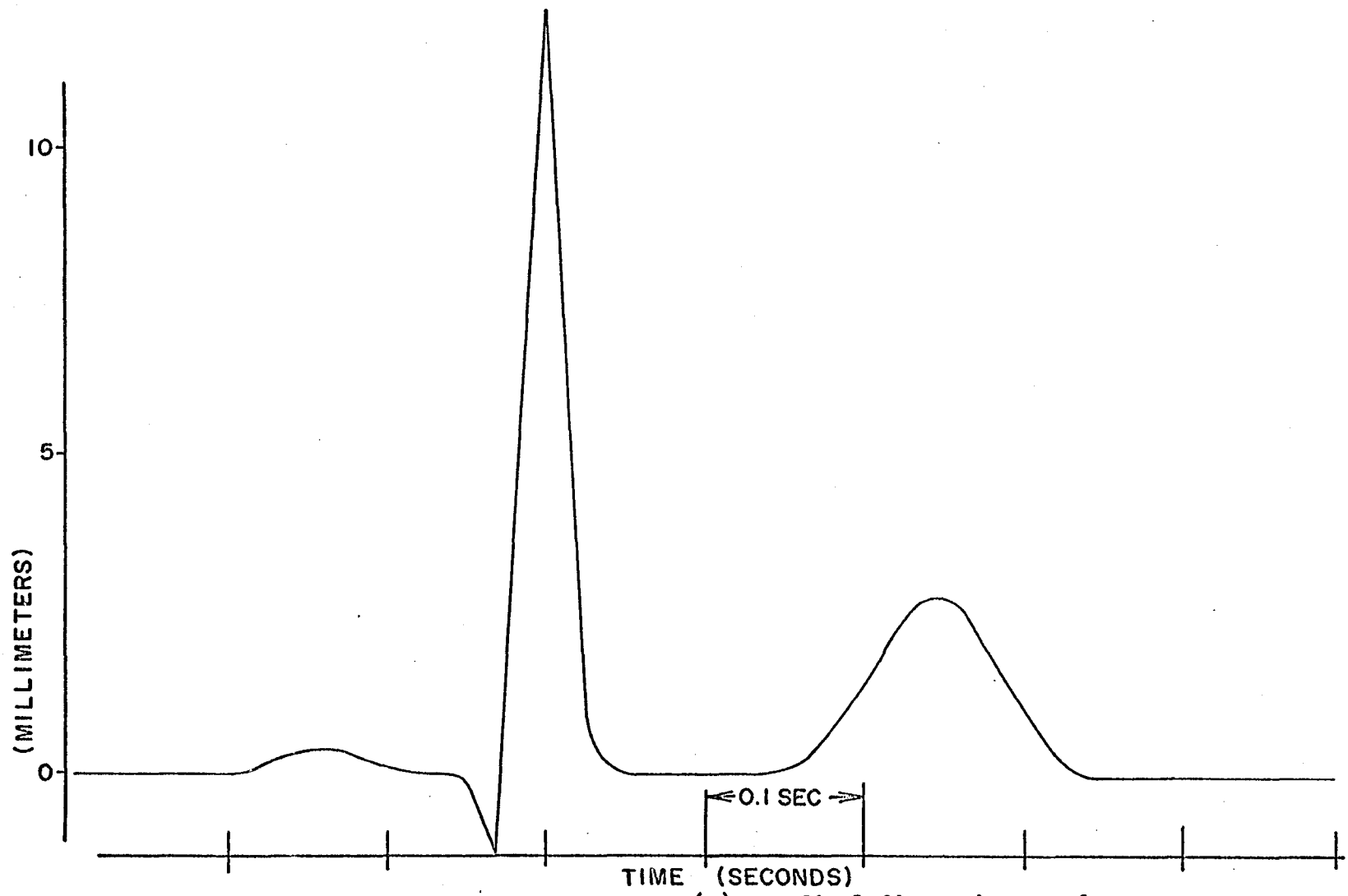


Figure 17. Test pattern $x_{35}(t)$ - medical diagnosis normal

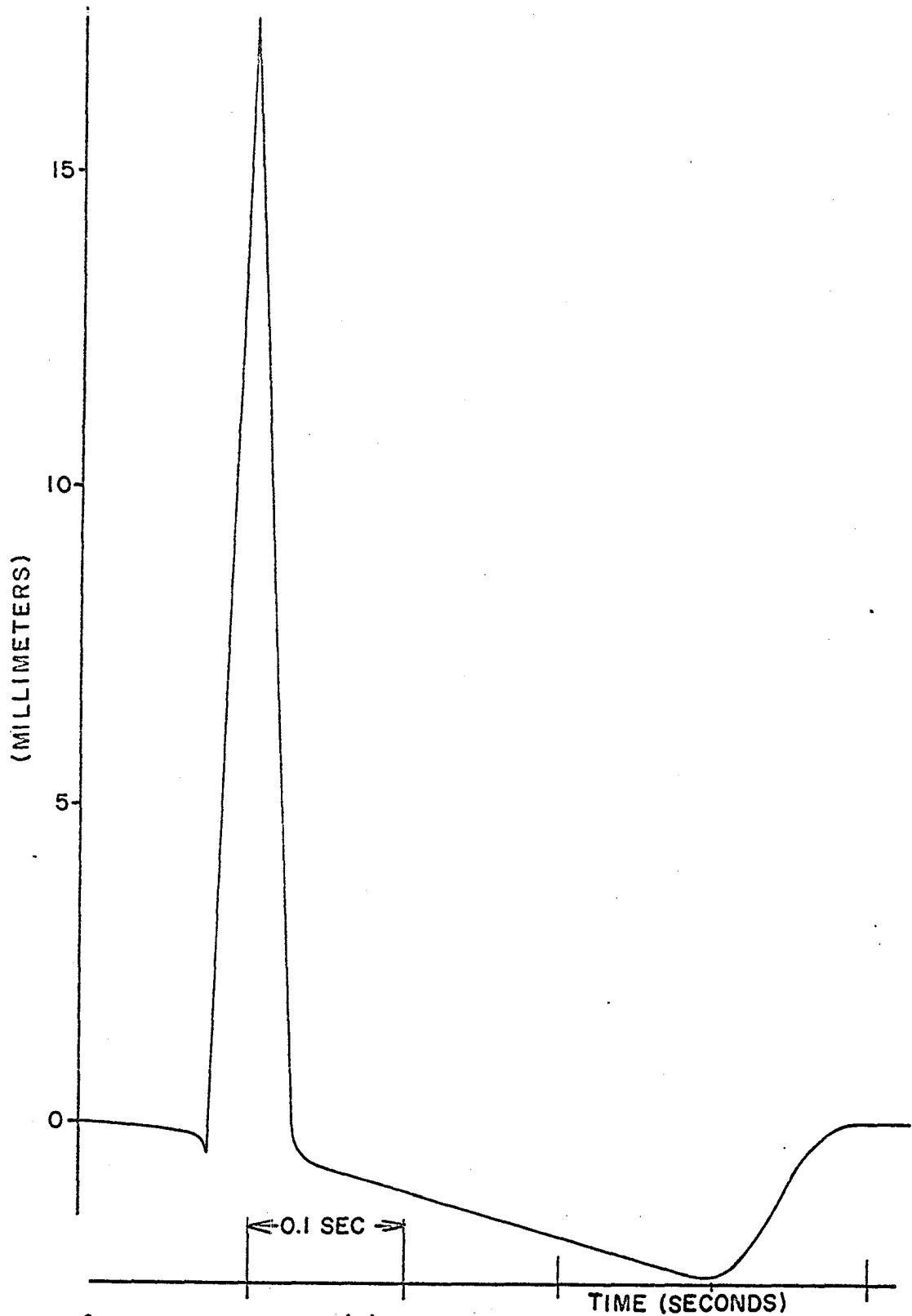


Figure 18. Test pattern $x_{14}(t)$ - medical diagnosis left ventricular hypertrophy

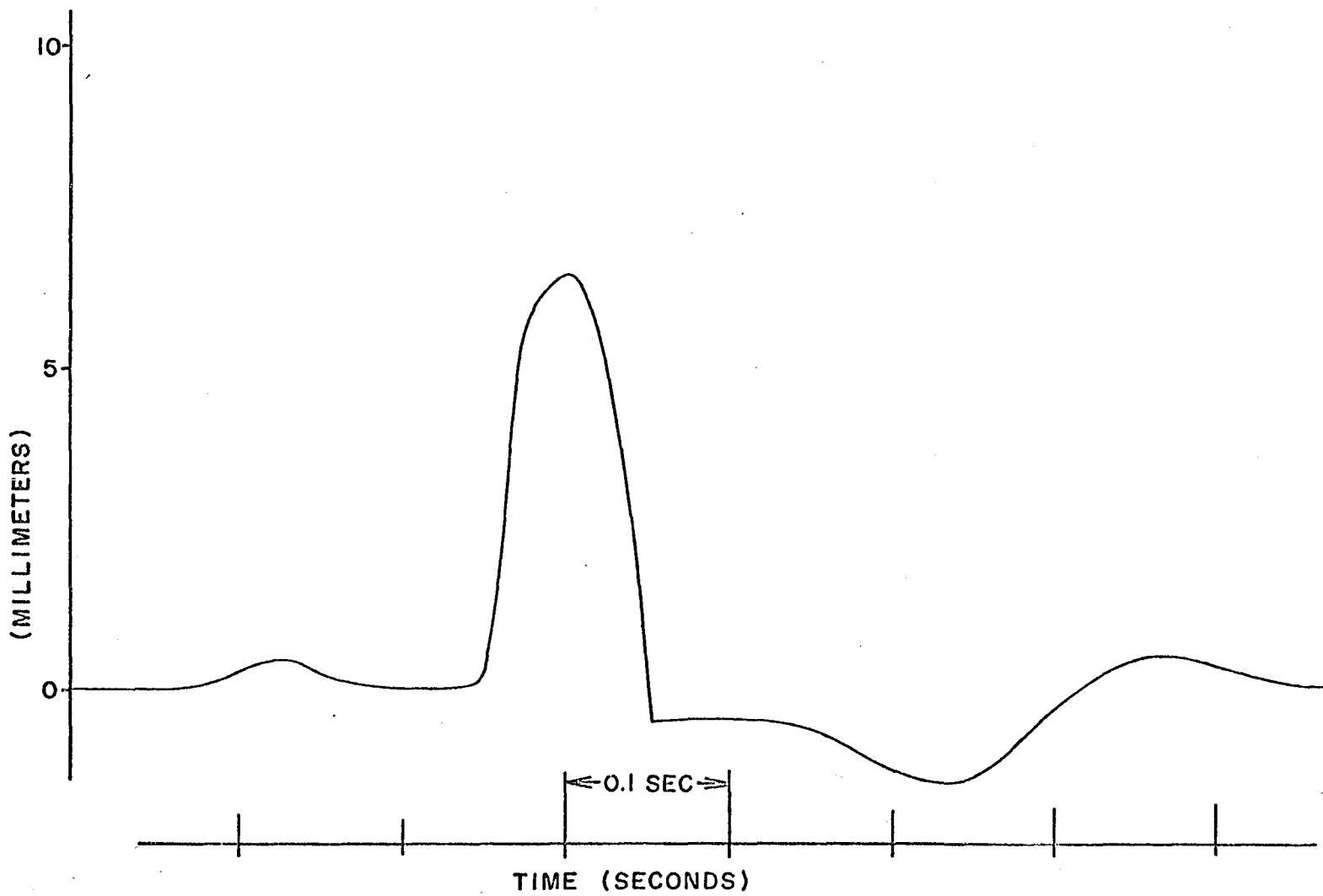


Figure 19. Test pattern $x_{21}(t)$ - medical diagnosis left bundle branch block

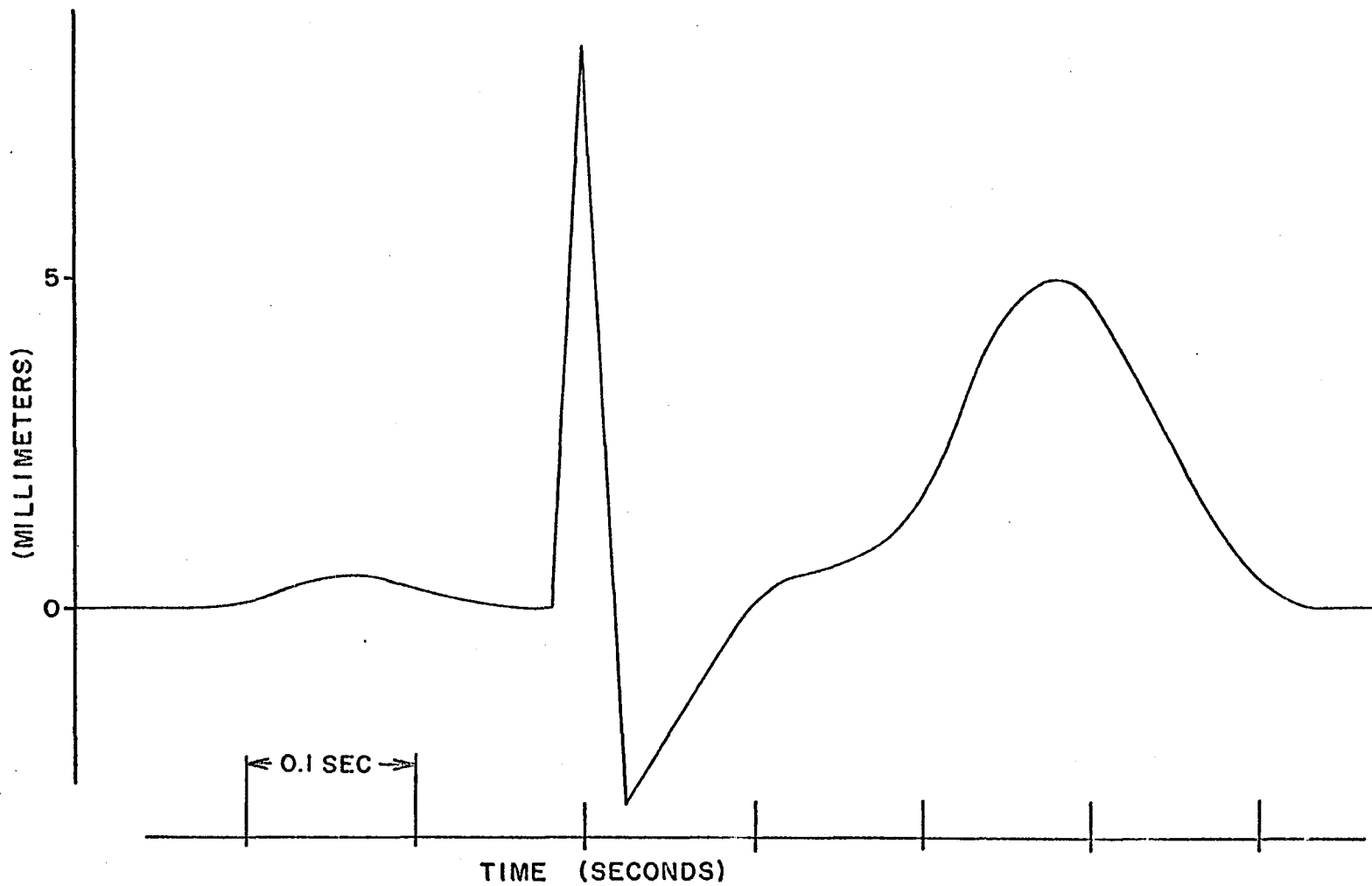


Figure 20. Test pattern $x_{13}(t)$ - medical diagnosis right ventricular hypertrophy

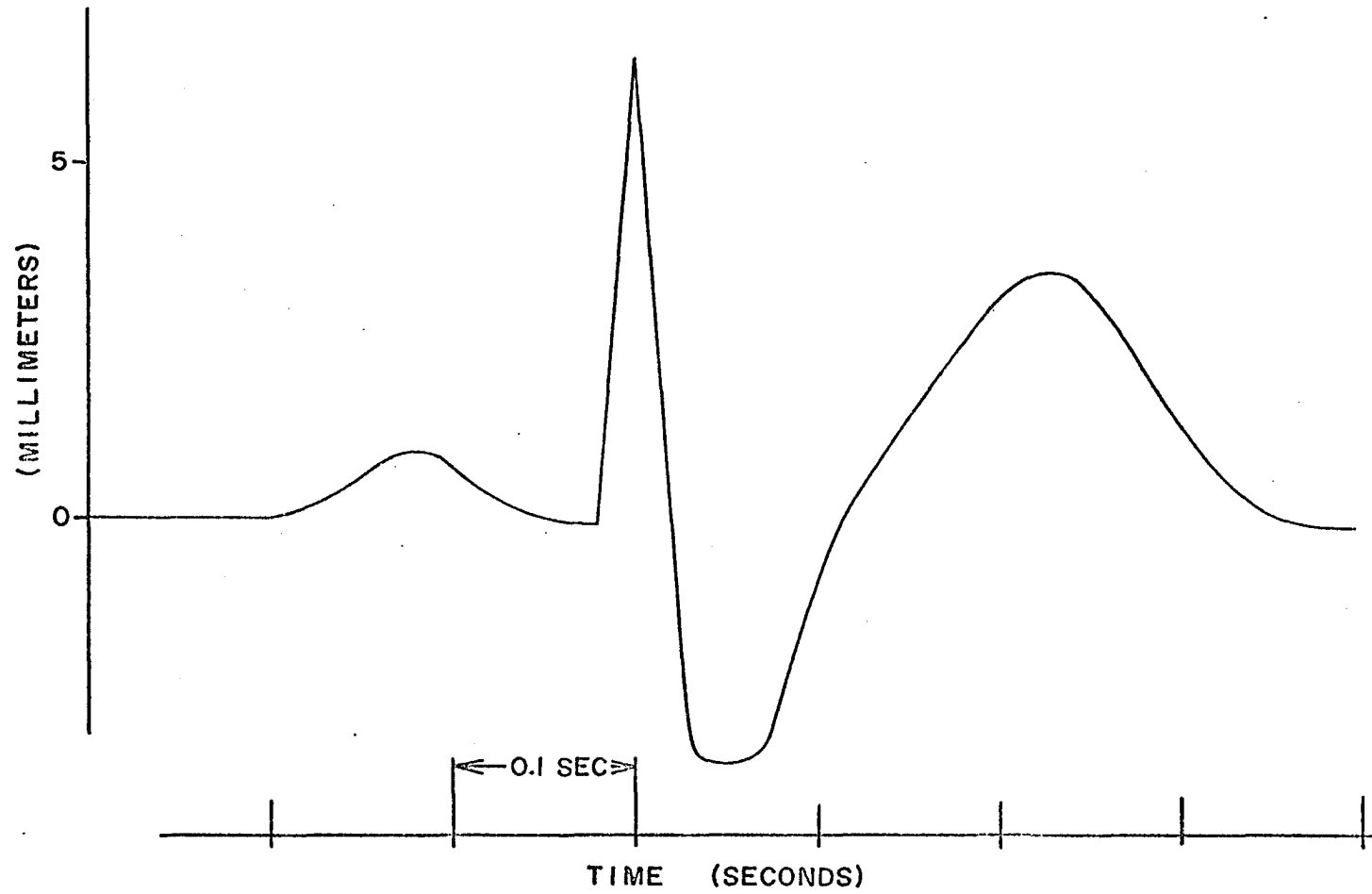


Figure 21. Test pattern $x_{52}(t)$ - medical diagnosis right bundle branch block

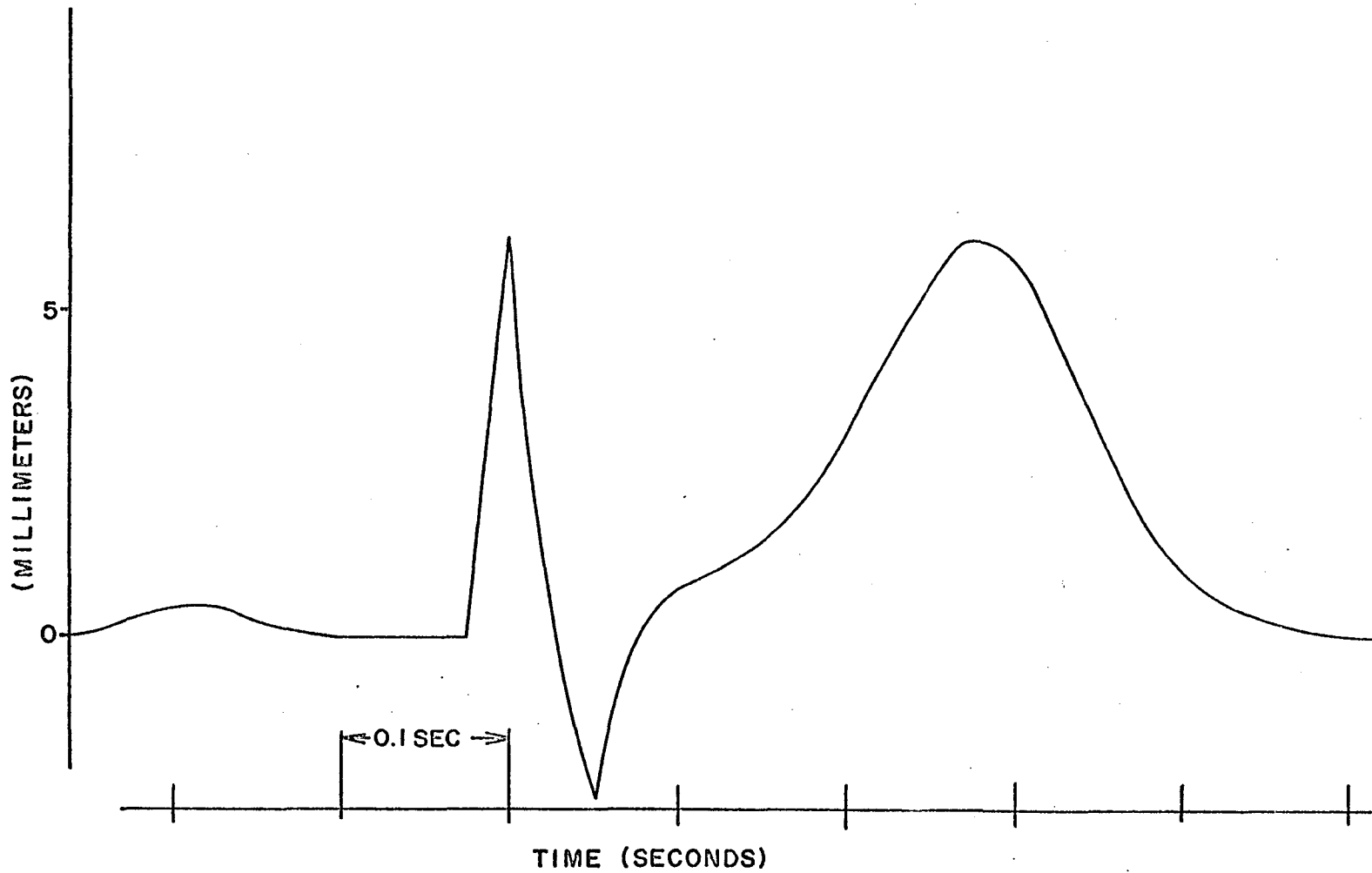


Figure 22. Test pattern $x_3(t)$ - medical diagnosis non-specific myocardial disease

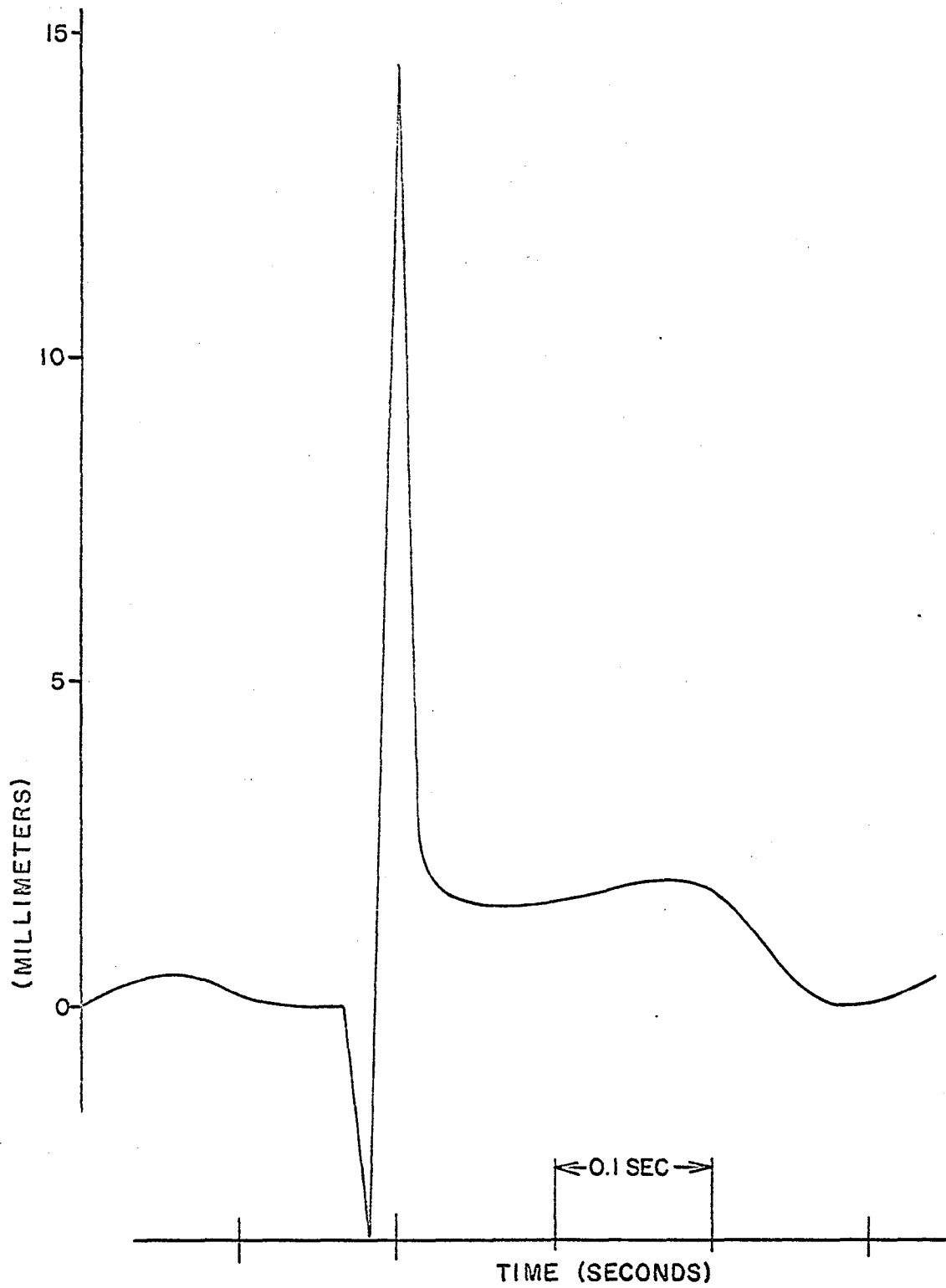


Figure 23. Test pattern $x_{40}(t)$ - medical diagnosis posterior myocardial infarction

Prior to obtaining the correlation coefficients of the various test waveforms when compared to selected standards, the machine program was checked on the digital computer (I.B.M. 1620) available for this study. Manual computation of the autocorrelation function of a sine wave was relatively easy but obtained an adequate check against the computer program. Analytical calculation of this wave yields an autocorrelation function for the sine wave $E \sin \omega t$ of $\frac{E^2}{2} \cos \omega t$. Values of $E = 100$ and $\omega = 2.5\pi$ radians per second were selected for the dummy standard pattern, i.e., the reference pattern was made $100 \sin (2.5\pi t)$. The frequency of the waveform is obviously equal to 1.25 cycles per second which implies that the cycle length is 0.8 second. This length corresponds to the cardiac cycle length of a heart rate of 75 beats per minute, which was the heart rate of the standard reference pattern for a normal heart. A dummy test pattern was chosen to be $50 \sin \frac{\omega}{2} t$ with $\omega = 2.5\pi$ radians per second as before. Again it is clear that this wave has a frequency of 0.625 cycles per second or is of duration 1.6 seconds. Thus, a time normalization as well as amplitude normalization was required. Sampling both the standard and the test pattern 81 times manually and then applying this data to the computer for amplitude normalization obtained an excellent pattern match. That is, the machine calculated result was 49.2 as compared to the manual analytical value of 50.0. On a per unit basis it may be stated that the test wave had a correlation coefficient of 0.985 or an error of approximately 1.5%. This error arises, of course, from the manual sampling to obtain the necessary data as well as the trapezoidal method of integration used. The manual measuring method used by cardiologists is approximately +5%, so the computer program accuracy is well within the

required limits.

As an aid in more clearly understanding what the normalization technique does, consider the waveform of Figure 24. Shown there is the diagrammatic representation of the cardiac cycle for a particular patient with a normal heart. It is seen that for the heart rate indicated of 64 beats per minute a cycle length of 0.94 sec results and that the amplitudes, in general, are much reduced from those of the standard pattern for a normal heart. After application of both time and amplitude normalization to the original waveform of Figure 24, the reshaped waveform is constructed and is shown in Figure 25. The cycle length is now 0.8 seconds in duration, corresponding to the standard pattern cycle length, and the amplitudes have all been modified by a factor of 2.81 to obtain the resultant wave shape. It may be observed that this normalized waveform more closely resembles the standard waveform selected for a reference as shown previously in Figure 9. The normalization technique has preserved, basically, the general shape of the individual test pattern. As a matter of interest a correlation coefficient of 0.96 was calculated in comparing this particular reconstructed waveform to the standard pattern for normal.

An indication of the results of the digital matched filter approach in making a diagnostic interpretation can be visualized by reference to Table 1. This table has listed the various test patterns or waveforms along with the medical diagnosis. Each test pattern has been correlated with the various standard or reference patterns and the resulting correlation coefficients are listed at the right. It will be noted that the reference patterns are distributed throughout the heart rate range of interest in this study. This was necessitated because of the limited

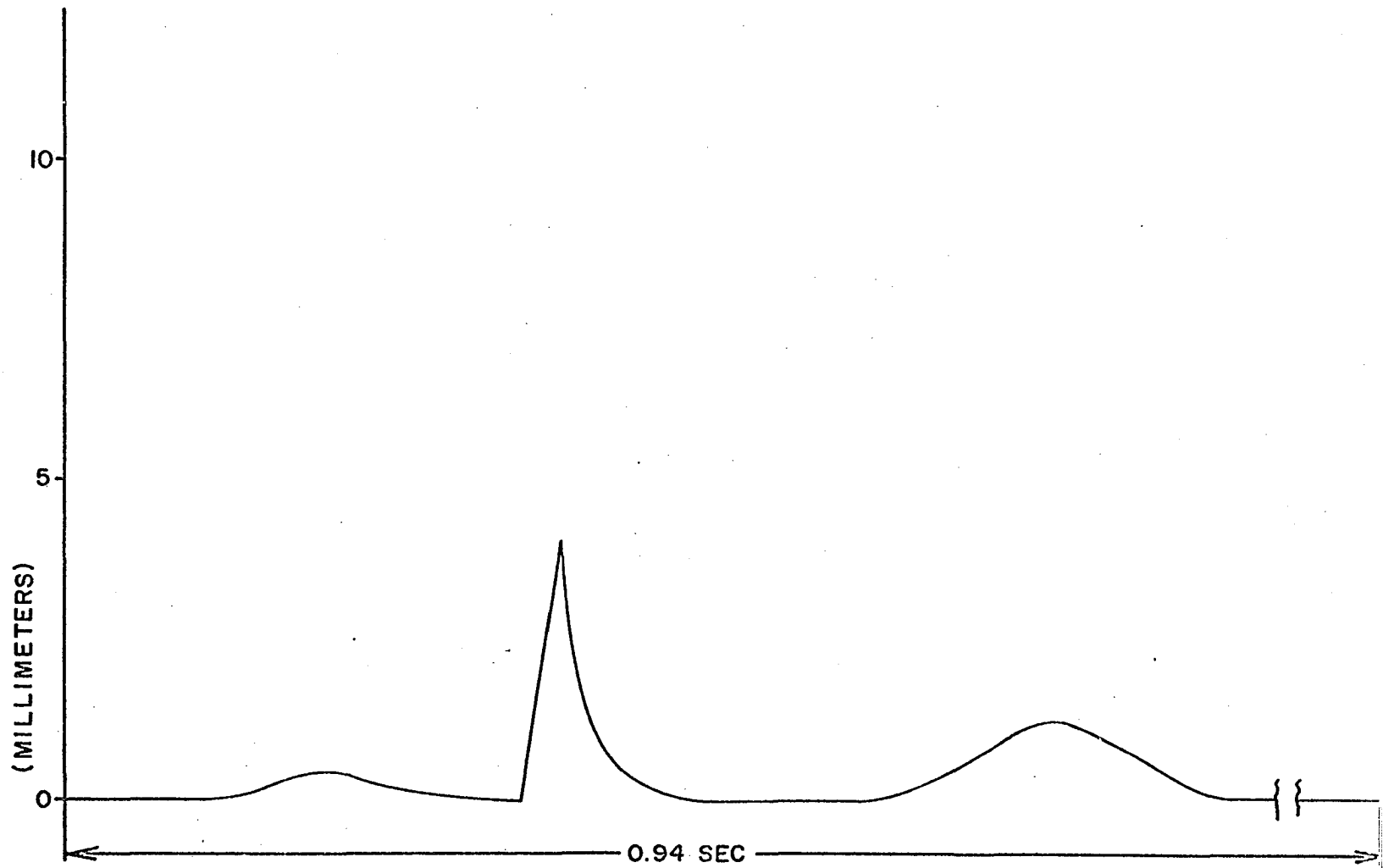


Figure 24. Test pattern $x_9(t)$ prior to normalization

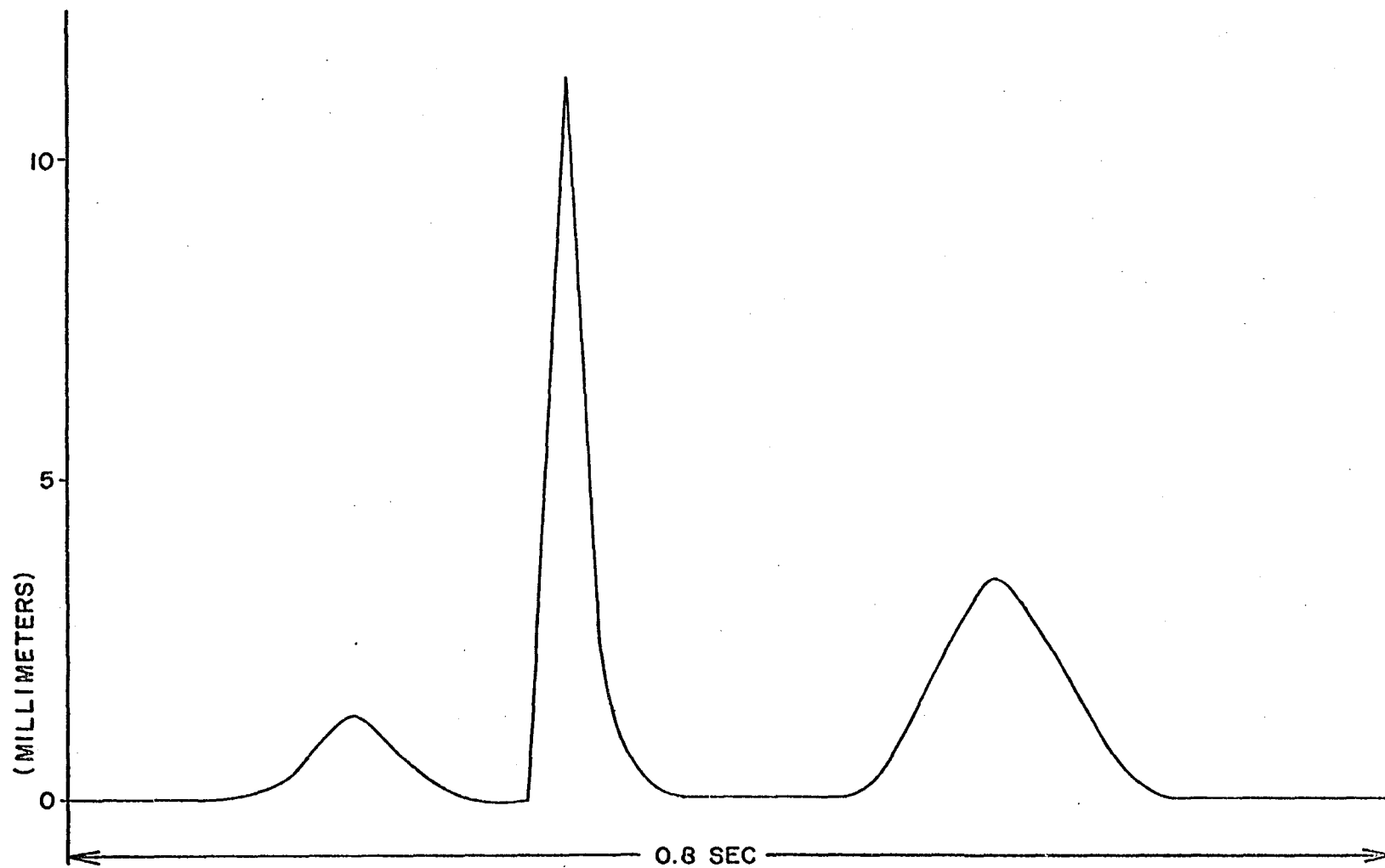


Figure 25. Test pattern $x_9(t)$ after being normalized

Table 1. Correlation of test patterns with selected standard patterns

Test Pattern	Heart Rate in Beats/Minute	Medical Diagnosis	Normal	Left Ventricular Hypertrophy
			$s_1(t)$	$s_2(t)$
$x_1(t)$	43	Non-specific myocardial disease	0.58	0.85
$x_2(t)$	51	Normal	0.80	0.62
$x_3(t)$	51	Non-specific auriculoventricular block	0.53	0.63
$x_4(t)$	52	Normal	0.83	0.57
$x_5(t)$	56	Normal	0.79	0.57
$x_6(t)$	57	Left ventricular hypertrophy	0.83	0.86
$x_7(t)$	58	Normal	0.84	0.72
$x_8(t)$	61	Left bundle branch block	0.45	0.85
$x_9(t)$	64	Normal	0.96	0.75
$x_{10}(t)$	65	Normal	0.93	0.72
$x_{11}(t)$	65	Normal	0.97	0.60
$x_{12}(t)$	65	Normal	0.96	0.58
$x_{13}(t)$	65	Right ventricular hypertrophy	0.84	0.56
$x_{14}(t)$	65	Left bundle branch block	0.39	0.93
$x_{15}(t)$	65	Left ventricular hypertrophy lateral infarction	0.67	0.93
$x_{16}(t)$	65	Normal	0.91	0.74
$x_{17}(t)$	66	Left ventricular hypertrophy	0.68	0.93
$x_{18}(t)$	67	Normal	0.91	0.64
$x_{19}(t)$	68	Normal	0.89	0.83
$x_{20}(t)$	68	Left ventricular hypertrophy	0.71	0.92
$x_{21}(t)$	68	Left bundle branch block	0.46	0.91
$x_{22}(t)$	68	Normal	0.93	0.79
$x_{23}(t)$	68	Normal	0.95	0.73
$x_{24}(t)$	70	Normal	0.95	0.53
$x_{25}(t)$	71	Left ventricular hypertrophy	0.67	0.95
$x_{26}(t)$	71	Left bundle branch block	0.53	0.80
$x_{27}(t)$	71	Normal	0.93	0.64
$x_{28}(t)$	71	Normal	0.98	0.66
$x_{29}(t)$	75	Normal	0.89	0.73
$x_{30}(t)$	75	Normal	0.85	0.85

Correlation of test patterns with standard patterns

Left Ventricular Hypertrophy	Left Bundle Branch Block	Right Ventricular Hypertrophy	Right Bundle Branch Block	Non-specific Auriculoventricular Block	Posterior Myocardial Infarction
$s_2(t)$	$s_3(t)$	$s_4(t)$	$s_5(t)$	$s_6(t)$	$s_7(t)$
0.85	0.66	0.30	0.48	0.36	0.48
0.62	0.50	0.60	0.44	0.70	0.93
0.63	0.74	0.73	0.56	1.00	0.65
0.57	0.50	0.81	0.60	0.74	0.88
0.57	0.55	0.77	0.58	0.77	0.88
0.86	0.65	0.56	0.58	0.46	0.82
0.72	0.54	0.69	0.56	0.67	0.93
0.85	0.79	0.39	0.52	0.47	0.29
0.75	0.56	0.78	0.71	0.54	0.87
0.72	0.58	0.78	0.71	0.54	0.82
0.60	0.44	0.85	0.72	0.58	0.87
0.58	0.54	0.89	0.75	0.60	0.83
0.56	0.64	1.00	0.76	0.74	0.65
0.93	0.79	0.42	0.33	0.50	0.65
0.93	0.72	0.36	0.49	0.44	0.65
0.74	0.57	0.77	0.67	0.60	0.91
0.93	0.79	0.46	0.44	0.55	0.70
0.64	0.46	0.84	0.69	0.66	0.87
0.83	0.67	0.63	0.61	0.56	0.91
0.92	0.78	0.47	0.44	0.56	0.77
0.91	1.00	0.64	0.57	0.74	0.55
0.79	0.66	0.70	0.73	0.58	0.84
0.73	0.57	0.80	0.72	0.55	0.88
0.53	0.38	0.89	0.74	0.64	0.82
0.95	0.80	0.47	0.40	0.57	0.76
0.80	0.92	0.74	0.57	0.85	0.55
0.64	0.58	0.82	0.72	0.56	0.82
0.66	0.51	0.75	0.78	0.55	0.83
0.73	0.59	0.74	0.64	0.64	0.92
0.85	0.72	0.62	0.59	0.64	0.90

Table 1 (Continued).

Test Pattern	Heart Rate in Beats/Minute	Medical Diagnosis	Normal	Le
			$s_1(t)$	
$x_{31}(t)$	75	Normal	0.86	
$x_{32}(t)$	75	Left ventricular hypertrophy	0.62	
$x_{33}(t)$	75	Normal	0.95	
$x_{34}(t)$	75	Normal	0.94	
$x_{35}(t)$	75	Normal	0.91	
$x_{36}(t)$	75	Normal	1.00	
$x_{37}(t)$	75	Normal	0.89	
$x_{38}(t)$	75	Normal	0.89	
$x_{39}(t)$	79	Normal	0.92	
$x_{40}(t)$	79	Normal	0.88	
$x_{41}(t)$	81	Normal	0.88	
$x_{42}(t)$	81	Myocardial infarction left ventricular hypertrophy	0.67	
$x_{43}(t)$	83	Normal	0.91	
$x_{44}(t)$	88	Normal	0.91	
$x_{45}(t)$	88	Normal	0.93	
$x_{46}(t)$	88	Myocardial infarction left ventricular hypertrophy	0.71	
$x_{47}(t)$	90	Normal	0.93	
$x_{48}(t)$	90	Normal	0.94	
$x_{49}(t)$	92	Posterior myocardial infarction	0.84	
$x_{50}(t)$	94	Left bundle branch block	0.39	
$x_{51}(t)$	94	Left ventricular hypertrophy	0.65	
$x_{52}(t)$	96	Right bundle branch block	0.77	
$x_{53}(t)$	100	Normal	0.90	
$x_{54}(t)$	100	Normal	0.95	
$x_{55}(t)$	110	Normal	0.83	

Left Ventricular Hypertrophy	Left Bundle Branch Block	Right Ventricular Hypertrophy	Right Bundle Branch Block	Non-specific Auriculoventricular Block	Posterior Myocardial Infarction
$s_2(t)$	$s_3(t)$	$s_4(t)$	$s_5(t)$	$s_6(t)$	$s_7(t)$
0.82	0.69	0.65	0.61	0.62	0.91
1.00	0.90	0.52	0.46	0.62	0.65
0.70	0.58	0.79	0.78	0.57	0.77
0.81	0.66	0.58	0.69	0.57	0.86
0.78	0.62	0.72	0.56	0.56	0.91
0.64	0.46	0.84	0.77	0.53	0.84
0.78	0.63	0.72	0.66	0.58	0.89
0.86	0.73	0.63	0.63	0.62	0.87
0.69	0.50	0.78	0.65	0.65	0.93
0.72	0.53	0.74	0.64	0.62	0.91
0.55	0.42	0.87	0.68	0.70	0.84
0.98	0.88	0.51	0.54	0.61	0.67
0.58	0.50	0.88	0.74	0.70	0.86
0.84	0.71	0.62	0.70	0.58	0.86
0.57	0.46	0.82	0.71	0.64	0.87
0.86	0.60	0.34	0.48	0.43	0.73
0.59	0.58	0.85	0.81	0.62	0.77
0.75	0.62	0.75	0.75	0.55	0.82
0.65	0.55	0.65	0.45	0.66	1.00
0.60	0.71	0.57	0.45	0.65	0.31
0.98	0.92	0.56	0.47	0.69	0.68
0.47	0.46	0.74	1.00	0.55	0.45
0.61	0.58	0.71	0.81	0.66	0.65
0.75	0.60	0.75	0.77	0.56	0.85
0.88	0.81	0.56	0.73	0.63	0.73

number of disease states other than normal which were available. A casual glance at Table 1 will indicate that relatively good or high correlation of the test patterns with those of the standard patterns appears to be somewhat random. More careful examination of the results are in order. Prior to this, it should be noted that all time normalization was handled on a strict 81 sample point basis, regardless of cycle length, to obtain the results listed in Table 1.

A careful analysis of Table 1 shows that excellent correlation of the test patterns is obtained for normal, left ventricular hypertrophy, and left bundle branch block. This result might have been anticipated since the patterns selected as reference for these disease states have heart rates within the median range of the test sample available. A relatively good correlation between left ventricular hypertrophy and left bundle branch block had been expected in this investigation.

Only one relatively distinct pattern was available for each of the other four selected reference patterns. Nevertheless some interesting observations can be made concerning the correlation of the test patterns with right ventricular hypertrophy, right bundle branch block and non-specific auriculoventricular block. For example, a relatively high correlation exists between right ventricular hypertrophy and right bundle branch block. Also the correlation between right ventricular hypertrophy and A-V block appears to be relatively good. This result was anticipated from consultation with a cardiologist. The correlation between infarction and the test patterns appears to be completely random and at this point of little value. Much more work will be necessary in developing possible

criteria for the above mentioned reference patterns, although the relatively low correlation of most of the disease states with the selected references offers some degree of encouragement. Premature judgements are certainly not in order until many more known patterns of these last four disease states are available.

Of primary interest in this investigation was the consideration of the possibility of one lead to screen for the normal or abnormal cardiac state. Studying Table 1 in some detail enables one to ascertain this possibility. It may be observed that the lowest correlation of a normal test pattern with that of the standard test pattern is 0.79. This affords some overlap with the correlation obtained for other of the disease states when compared with normal. Right ventricular hypertrophy correlates to 0.84, for example, with the normal standard pattern. In general, however, normal test patterns correlate very well with the normal standard pattern. As a matter of fact, one medical diagnosis was changed, as has been indicated earlier, from left ventricular hypertrophy to normal after considerable progress had been made on the machine study. Three different standard patterns for normal were being utilized which convinced this author of a possible error in medical diagnosis. Consultation with a cardiologist confirmed the suspicion that $X_{16}(t)$ should be interpreted as normal. Needless to say this was very satisfying in and of itself.

Application of a correction to the time normalization yields a quite satisfactory discrimination of the normal test patterns throughout the heart rate range. Prior to this correction it was observed that satisfactory discrimination was achieved over a considerable heart rate range. From a heart rate of 60 to a heart rate of 100, a threshold value of 0.85

discriminated against disease states other than normal. Thus, after the time normalization correction, for heart rates less than 60 and greater than 100, one obtains the results indicated in Table 2. It is now clearly seen that all normals but X_2 can be detected by requiring a threshold value of 0.85 throughout the entire heart rate range. In the succeeding discussion the applicability of these results will be examined further.

Prior to discussing the applicability of the above results, it is of interest to note what one obtains by highest correlation as a criterion rather than the threshold value method. This requires that only the highest correlation value obtained with cataloged standards in Table 1 for each test waveforms is necessary in a machine decision. On this basis it is determined that a correct computer decision can be made 65 per cent of the time. This appears to be surprisingly poor. As mentioned previously, however, only one relatively distinct pattern was available for each of the last four reference patterns. It was noted that the correlation between infarction and the test patterns appeared to be random. By disregarding standard pattern 7, it can be ascertained that a correct machine decision is possible for 34 of 36 normals and 15 of 19 of the remaining disease states. This is a decided improvement but emphasizes an earlier remark indicating that much work is necessary in developing criteria for reference patterns.

Table 2. Correlation of time corrected test patterns with selected standard patterns

Test Pattern	Heart Rate in Beats/Minute	Medical Diagnosis	Correlation of test patterns with standard patterns		
			Normal $s_1(t)$	LVH $s_2(t)$	LBB $s_3(t)$
$x_1(t)$	43	Non-specific myocardial disease	0.47	0.90	0.78
$x_2(t)$	51	Normal	0.81	0.64	0.53
$x_3(t)$	51	Non-specific myocardial disease	0.69	0.57	0.70
$x_4(t)$	52	Normal	0.96	0.62	0.50
$x_5(t)$	56	Normal	0.90	0.62	0.54
$x_6(t)$	57	Left ventricular hypertrophy	0.83	0.88	0.67
$x_7(t)$	58	Normal	0.91	0.80	0.64
$x_{55}(t)$	110	Normal	0.88	0.82	0.70

IV. DISCUSSION

A. Practical Considerations in Application of Results

From the preceding development it has been ascertained that it is possible to make a discriminate choice of a normal heart pattern as obtained from an electrocardiograph. Careful selection of a reference pattern was of particular importance. Due regard for typical intervals associated with the QRS complex and that of the T wave was mandatory in this selection. Since the criterion for discrimination was based on area or more specifically the square of an area, the P wave was of little significance in this diagnostic technique. Through application of a time correction on test patterns with respect to normalization it appears feasible to use one reference or standard pattern for normal throughout the heart rate range studied. A much more comprehensive machine study is necessary to validate this assumption, however.

A careful examination of the results obtained for left ventricular hypertrophy and left bundle branch block indicates a high percentage of "recognition" for these disease states. It appears reasonable that one could expect to make a preliminary diagnosis for these diseases by machine. For practical application, however, it is more conceivable to anticipate a preliminary screening for the normal cardiovascular state by the computer. After such screening any abnormal possibilities would likely be referred to the cardiologist for further consultation.

As mentioned earlier, one reference pattern obtained respectable results over the complete range of heart rates for the cases utilized in this study. Logically, a number of reference patterns could be obtained

for machine use. Each pattern would be the reference over a fixed heart rate range. By storing these in memory within the computer for screening purposes, no correction on time normalization would be anticipated. It is questionable, however, that this could affect any savings of time in the machine diagnosis, particularly if manual control by a technician of the sampling rate is realized.

A possible system for facilitating the method of screening as developed by this dissertation is shown in Figure 26. Briefly, this system would consist of a method of recording the data on magnetic tape, a preliminary smoothing and then differentiating the signal by analog methods, converting the output to digital form, and application of a logic of recognition digitally to determine the base line and RR interval of the electrocardiogram. Since the determination of RR interval obtains the required data for heart rate, any time normalization corrections could be realized digitally. The ECG preamplifier, tape recorder (two channel), direct writer, and the digital coder included would be assembled into one mobile unit. This would afford easy adaptation to the usual hospital routine. Also indicated for this system is a means of direct on-line operation as a method of supplementing the research effort required for such a project. The on-line system is essentially a duplication of that discussed above except for the analog tape requirement.

A group from Oklahoma City has proposed a one lead screening electrocardiogram for medical utilization. The use of this technique, under the methods proposed in the preceding discussion, would seem to be the solution to the problem of routine electrocardiograms on all hospital admissions. The increased incidence of heart disease in our population requires an

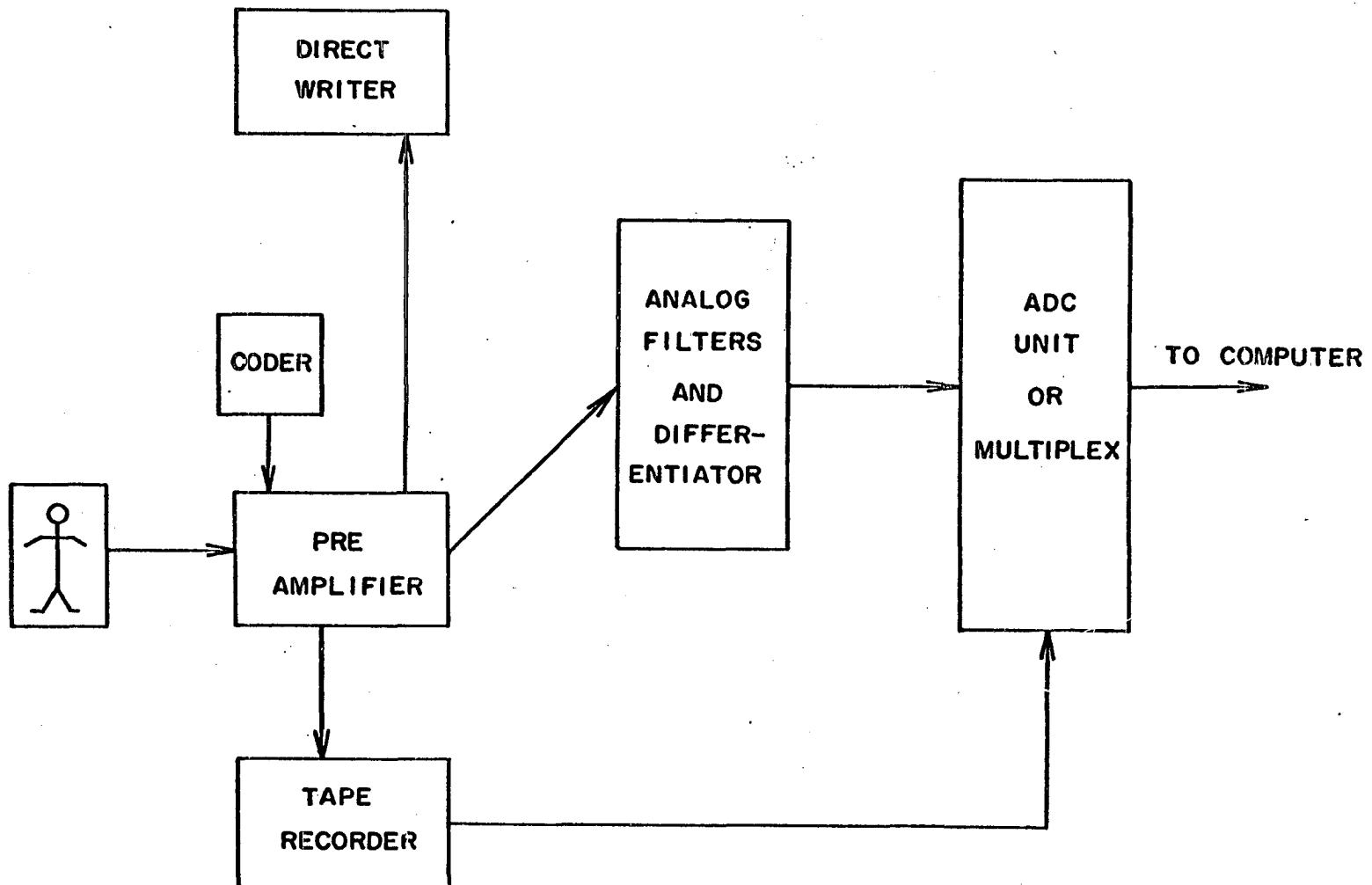


Figure 26. Possible automatic electrocardiographic diagnostic system

increasing alertness to this disease in patients admitted to a hospital for other problems. It is likely that an electrocardiogram on each admission would pick up a number of unknown cases of heart disease. Since, the cost in time and energy of a standard screening electrocardiogram is presently impractical it seems entirely feasible to incorporate a digital computer system to accomplish this project.

B. Comparison with Other Techniques

The only known operative application for automatic pattern recognition in the clinical electrocardiogram is that undertaken by the National Institute of Health in Washington, D. C. This effort was undertaken to demonstrate the feasibility of the use of computers in extracting clinically useful parameters from electrophysiologic waveforms. Criteria for clinically significant voltage fluctuations of the signal from the baseline with specified time intervals were determined. The computer was programmed to identify those fluctuations automatically. For an output, the computer produces a set of measurements of ECG waveforms from one cardiac cycle in any random 5 second portion of a lead. The program permits determination of amplitude of P, Q, R, S, and T waves, ST and PQ segments, and QT and RR intervals. These measurements are obtained at present on all 12 of the usually recorded leads associated with the clinical electrocardiogram. The logic of recognition has correctly measured 767 out of 770 parameters in the ECG's tested. At this writing, it has been reported verbally to the author that this system utilizing all twelve leads, was now capable of 100 percent recognition of the normal ECG.

The Department of Medicine, University of Oklahoma Medical Center, in

Oklahoma City, Oklahoma has conducted an intensive research effort into developing a new single lead electrocardiogram. This group headed by Linderman (20) has pointed out that the finding of significant electrocardiographic abnormalities often makes it possible to recognize heart disease which is undetectable by other techniques, such as chest X-ray examination and physical examination of the patient. Simple, rapid, and reliable electrocardiographic screening techniques are needed for surveys of mass population because the multiple-lead electrocardiogram is generally too time consuming and expensive for large scale screening. A single oblique electrocardiographic lead which makes it possible to detect a high percentage of significant electrocardiographic abnormalities in a large number of subjects with minimum effort and maximum convenience has been studied. This single lead technique was tested on 996 subjects and compared with standard electrocardiograms in order to check the validity of the single lead as a screening technique. The oblique chest lead was effective in demonstrating the screening possibilities for detection of abnormal heart disease. In the initial studies a sensitivity of 83 percent was obtained, where sensitivity of the screening lead was defined as the percentage of abnormal single-lead electrocardiograms confirmed by the standard tracing compared to the total number of electrocardiographic abnormalities determined by the standard tracing. Electrocardiograms read as abnormal or suspiciously abnormal by the single lead constituted referral of the subject for a complete electrocardiogram. The percentage of normal single-lead electrocardiograms confirmed by the standard electrocardiogram compared to the total number of normal electrocardiograms determined by the standard electrocardiogram was defined as specificity. Initially a

specificity of 87 percent was obtained in this study. With further experience, sensitivity and specificity rates of about 90 percent was obtained.

Utilization of a single lead screening technique, as discussed above, shows considerable promise for practical application. In the application above it is noted that all records were read manually to check for certain prescribed amplitudes and intervals of the patterns. It is apparent that a single lead system could be easily incorporated into a computer system to be detected automatically via the standard clinical parameters. Of immediate interest, in so far as this dissertation is concerned; is the fact that by correlation techniques for the 36 single lead, normal patterns, studied by the author, correct detection of normal was obtained in 35 cases. Detection of abnormal may be obtained in all 19 abnormal patterns studied. Although the sample studied for this dissertation is small compared to the sample obtained by the group in Oklahoma City, it appears evident that the correct screening for heart disease is at least comparable on a percentage basis.

V. SUMMARY

This dissertation has reported on the results of one investigation being conducted in cooperation with Iowa Methodist Hospital. The specific notion was that of making an automatic diagnosis of heart disease through the use of a digital computer. An investigation of the use of correlation techniques in the diagnosis of a cardiac disease state was achieved through utilization of the entire waveform of the electrocardiogram. By using the complete waveform of one lead, in contrast to the usual twelve leads recorded, it has been shown that automatic screening for heart disease is a distinct possibility.

Basically, a matched filter concept was applied to the recognition problem associated with the pattern of the clinical electrocardiogram. It has been shown that this obtains an output as a function of real time which is the autocorrelation function of the waveform. Since sequential samples are dealt with, the computation of the correlation function lends itself readily to machine methods.

A normalization technique was required for each pattern that entailed, basically, a concept of preserving to a considerable extent the general shape of each individual pattern. It was found that these methods yielded good results over a wide range of heart rates for the records studied. From the results obtained by the methods proposed in this dissertation, it was suggested that this technique is applicable to a one lead screening electrocardiogram for all hospital admissions.

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